Incorporating Objective Function Information into the Feasibility Rule for Constrained Evolutionary Optimization

Yong Wang, *Member, IEEE*, Bing-Chuan Wang, Han-Xiong Li, *Fellow, IEEE*, and Gary G. Yen, *Fellow, IEEE*

***Abstract*—When solving constrained optimization problems by evolutionary algorithms, an important issue is how to balance constraints and objective function. This paper presents a new method to address the above issue. In our method, after generating an offspring for each parent in the population by making use of differential evolution, the well-known feasibility rule is used to compare the offspring and its parent. Since the feasibility rule prefers constraints to objective function, the objective function information has been exploited as follows: if the offspring cannot survive into the next generation and if the objective function value of the offspring is better than that of the parent, then the offspring is stored into a predefined archive. Subsequently, the individuals in the archive are used to replace some individuals in the population according to a replacement mechanism. Moreover, a mutation strategy is proposed to help the population jump out of a local optimum in the infeasible region. Note that, in the replacement mechanism and the mutation strategy, the comparison of individuals is based on objective function. In addition, the information of objective function has also been utilized to generate offspring in differential evolution. By the above processes, this paper achieves an effective balance between constraints and objective function in constrained evolutionary optimization. The performance of our method has been tested on two sets of benchmark test functions, namely, 24 test functions at IEEE CEC2006, and 18 test functions with 10 and 30 dimensions at IEEE CEC2010. The experimental results have demonstrated that our method shows better or at least competitive performance against other state-of-the-art methods. Furthermore, the advantage of our method increases with the increase of the number of decision variables.**

***Index Terms*—Constrained optimization problems, constraints, evolutionary algorithms, objective function.**

I. Introduction

Without loss of generality, constrained optimization problems (COPs) can be formulated as follows:

$$\begin{aligned} &\text{minimize} \quad f(\vec{x}),\ \vec{x}=(x_1,\ldots,x_D)\in S,\ L_i\le x_i\le U_i \\ &\text{subject to:} \quad g_j(\vec{x})\le 0, \quad j=1,\ldots,l \\ &\qquad\qquad\quad h_j(\vec{x})=0, \quad j=l+1,\ldots,m \end{aligned}$$

where $\vec{x}$ is the decision vector, x_i is the ith decision variable, L_i and U_i are the lower and upper bounds of x_i, respectively, D is the number of decision variables, $S=\prod_{i=1}^{D}[L_i,U_i]$ is the decision space, $f(\vec{x})$ is the objective function, $g_j(\vec{x})$ is the jth inequality constraint, $h_j(\vec{x})$ is the (j-l)th equality constraint, l is the number of inequality constraints, and $(m-l)$ is the number of equality constraints.

For COPs, the degree of constraint violation of a decision vector $\vec{x}$ on the jth constraint is computed via the following equation:

$$G_j(\vec{x})=\begin{cases}\max\{0,g_j(\vec{x})\}, & 1\le j\le l\\ \max\{0,|h_j(\vec{x})|-\delta\}, & l+1\le j\le m\end{cases} \tag{1}$$

where δ is a positive tolerance value to relax equality constraints to a certain extent. Then, $G(\vec{x})=\sum_{j=1}^{m}G_j(\vec{x})$ represents the degree of constraint violation of $\vec{x}$ on all constraints. In the context of equation (1), a decision vector $\vec{x}$ is called a feasible solution if $G(\vec{x})=0$, otherwise $\vec{x}$ is called an infeasible solution. The decision space of a COP is composed of the feasible region and the infeasible region. The former is the set of all feasible solutions and the latter is the set of all infeasible solutions.

In the field of evolutionary computation, there has been a growing interest in applying evolutionary algorithms (EAs) to solve COPs. Due to the presence of constraints, many constrain-handling techniques have been suggested and integrated with EAs, and as a result, a variety of constrained optimization EAs (COEAs) have been proposed [1]-[3]. The current popular constraint-handling techniques can be briefly classified into three categories: methods based on penalty functions [4]-[6], methods based on the preference of feasible solutions over infeasible solutions [7]-[12], and methods based on multiobjective optimization [13]-[19]. Actually,

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Y. Wang is with the School of Information Science and Engineering, Central South University, Changsha 410083, China, and also with the Department of Systems Engineering and Engineering Management, City University of Hong Kong, Hong Kong. (e-mail: ywang@csu.edu.cn)

B.-C. Wang is with the School of Information Science and Engineering, Central South University, Changsha 410083, China. (e-mail: bcwang@csu.edu.cn)

H.-X. Li is with the Department of Systems Engineering and Engineering Management, City University of Hong Kong, Hong Kong, and also with the State Key Laboratory of High Performance Complex Manufacturing, Central South University, Changsha 410083, China. (e-mail: mehxli@cityu.edu.hk)

G. G. Yen is with the School of Electrical and Computer Engineering, Oklahoma State University, Stillwater, OK 74078 USA. (e-mail: gyen@okstate.edu)

after producing an offspring population for the parent population by EAs, the purpose of constraint-handling techniques is to determine a criterion to compare the individuals in the parent and offspring populations. The methods based on penalty functions construct a fitness function by adding a penalty term proportional to the constraint violation into objective function, and then uses this fitness function to compare the individuals. In the methods based on the preference of feasible solutions over infeasible solutions, the comparison of individuals is based on either the degree of constraint violation or objective function. Moreover, feasible solutions are always considered to be better than infeasible ones to a certain degree. In addition, the methods based on multiobjective optimization transform a COP into a multi-objective optimization problem with two objectives (i.e., $(f(\vec{x}),G(\vec{x}))$) or a multiobjective optimization problem with (m+1) objectives (i.e., $(f(\vec{x}),G_1(\vec{x}),\ldots,G_m(\vec{x}))$). After the above transformation, Pareto dominance is usually employed to compare the individuals.

In general, COEAs have two main tasks: 1) entering the feasible region rapidly, and 2) finding the optimal solution at the end. In order to accomplish the first task, the comparison of individuals is dependent mainly on constraints in most of constraint-handling techniques. As a result, the information of objective function has been neglected unreasonably, which has a negative effect on the achievement of the second task.

Motivated by the above consideration, this paper proposes a new COEA. We call this approach as the *f*easibility *r*ule with the incorporation of *o*bjective *f*unction *i*nformation (FROFI). In FROFI, differential evolution (DE) [20] serves as the search engine and the well-known feasibility rule [7] is used to compare the individuals in the population. During the evolution, if an offspring generated by DE is worse than the parent according to the feasibility rule and if the offspring has a better objective function value than its parent, the offspring will be stored into a predefined archive. Afterward, the individuals in the archive are used to replace some individuals in the population by a replacement mechanism. In addition, a mutation strategy is proposed. It is noteworthy that the comparison of individuals is based on objective function in both the replacement mechanism and the mutation strategy. Moreover, the information of objective function has also been used to guide the search in DE.

The main contributions of this paper can be summarized as:

- Due to the fact that the feasibility rule prefers constraints to objective function, FROFI incorporates the objective function information into the feasibility rule by three processes, i.e., the DE operators, the replacement mechanism, and the mutation strategy. The purpose of the DE operators is to balance the exploration and exploitation abilities of FROFI. The replacement mechanism is able to diversify the population at the early stage of evolution and enhance the convergence speed at the middle and later stages of evolution. In addition, the mutation strategy aims at alleviating premature convergence in the infeasible region. By the above three processes, overall, FROFI reaches a reasonable tradeoff between constraints and objective function.
- Systematic experiments have been conducted to compare FROFI with other well-established methods on two sets of benchmark test functions, namely, 24 test functions at IEEE CEC2006 [21], and 18 test functions with 10 and 30 dimensions at IEEE CEC2010 [22]. The experimental results have indicated that FROFI is better than or at least comparable to other methods and has good scalability to the number of decision variables. Moreover, FROFI has also been applied to solve constrained mechanical design optimization problems and constrained multi-objective optimization problems.
- The effectiveness of the replacement mechanism and the mutation strategy and the sensitivity of the parameter associated with the replacement mechanism have been experimentally investigated.

The rest of this paper is organized as follows. Section II introduces DE and the feasibility rule. Section III describes the related work. The proposed method, FROFI, is elaborated in Section IV. Section V presents the performance analysis and comparison, and more experimental studies on FROFI. Finally, Section VI concludes this paper.

II. Differential Evolution and the Feasibility Rule

A. Differential Evolution

Differential evolution (DE) was proposed by Storn and Price [20] in 1995. Like other EA paradigms, DE is a population-based optimization method. The population of DE can be expressed as follows:

$$P_t=\{\vec{x}_{1,t},\ldots,\vec{x}_{NP,t}\} \tag{2}$$

where t is the generation number, NP is the population size, and $\vec{x}_{i,t}=(x_{i,1,t},\ldots,x_{i,D,t})$ is the ith individual. In DE, $\vec{x}_{i,t}(i\in\{1,\ldots,NP\})$ is also called a target vector.

DE includes three main evolutionary operators: mutation, crossover and selection.

Mutation: The mutation operator creates a mutant vector for each target vector through utilizing the differential information of pairwise individuals. The following two mutation operators are adopted in this paper:

- DE/current-to-rand/1:

$$\vec{v}_{i,t}=\vec{x}_{i,t}+F\cdot(\vec{x}_{r1,t}-\vec{x}_{i,t})+F\cdot(\vec{x}_{r2,t}-\vec{x}_{r3,t}) \tag{3}$$

- DE/rand-to-best/1:

$$\vec{v}_{i,t}=\vec{x}_{r1,t}+F\cdot(\vec{x}_{best,t}-\vec{x}_{r1,t})+F\cdot(\vec{x}_{r2,t}-\vec{x}_{r3,t}) \tag{4}$$

where $i=1,\ldots,NP$, $r1$, $r2$, and $r3$ are mutually different integers randomly chosen from $[1,NP]\setminus i$, $\vec{x}_{best,t}$ is the best individual in the current population, F is the scaling factor, and $\vec{v}_{i,t}=(v_{i,1,t},\ldots,v_{i,D,t})$ is the mutant vector.

Crossover: DE performs a crossover operator on the target vector $\vec{x}_{i,t}$ and its mutant vector $\vec{v}_{i,t}$ to generate the trial vector $\vec{u}_{i,t}=(u_{i,1,t},\ldots,u_{i,D,t})$. The binomial crossover is implemented as follows:

$$u_{i,j,t} = \begin{cases} v_{i,j,t}, & \text{if } rand_j \le CR \text{ or } j = j_{rand} \\ x_{i,j,t}, & \text{otherwise} \end{cases} \quad (5)$$

where $i = 1,...,NP$, $j = 1,...,D$, $rand_j$ is a uniformly distributed random number on the interval [0,1] and regenerated for each j, j_{rand} is an integer randomly chosen from $[1, D]$, and CR is the crossover control parameter.

Selection: The target vector $\vec{x}_{i,t}$ is compared with its trial vector $\vec{u}_{i,t}$, and the better one will be selected for the next generation:

$$\vec{x}_{i,t+1} = \begin{cases} \vec{u}_{i,t}, & \text{if } f(\vec{u}_{i,t}) \le f(\vec{x}_{i,t}) \\ \vec{x}_{i,t}, & \text{otherwise} \end{cases} \quad (6)$$

B. The Feasibility Rule

The feasibility rule proposed by Deb [7] serves as the constraint-handling technique in this paper. This rule belongs to the methods based on the preference of feasible solutions over infeasible solutions mentioned in Section I, which compares pairwise individuals as follows:

1) Between two infeasible solutions, the one with smaller degree of constraint violation is preferred;
2) If one solution is infeasible and the other one is feasible, the feasible solution is preferred;
3) Between two feasible solutions, the one with better objective function value is preferred.

III. The Related Work

Recent two decades have witnessed significant progress in the development of EAs for COPs. In 2011, Mezura-Montes and Coello Coello [3] carried out a comprehensive survey on constraint-handling in nature-inspired numerical optimization. Recent developments during the last four years are briefly outlined below.

1) *Methods based on penalty functions*: Kusakci and Can [23] integrated a modified covariance matrix adaptation evolution strategy (CMA-ES) [24] with a penalty approach introduced in [4]. de Melo and Iacca [25] modified the stopping criteria and the sampling mechanism of CMA-ES [24], and introduced an adaptive penalty function into the modified CMA-ES. Ali and Zhu [26] proposed a constrained DE equipped with penalty function. Moreover, they provided theoretical results about the setting of the penalty coefficient. Hernández *et al.* [27] proposed a hybridization of DE and hill climbing, which employs static penalty to deal with constraints. In [28], a rough penalty method inspired by Pawlak's rough set theory [29] is proposed and coupled with an improved genetic algorithm. Li and Zhang [30] proposed an interesting piece of work, in which the minimum penalty coefficient is estimated at each generation. Wang and Cai [31] proposed (μ+λ)-CDE, which divides the evolutionary process into three situations: infeasible situation, semi-feasible situation, and feasible situation. In the semi-feasible situation, an adaptive penalty function is devised. Based on the constraint-handling framework in [31], Gong *et al.* [32] proposed two improvements: a ranking-based mutation operator of DE and a dynamic diversity mechanism.

2) *Methods based on the preference of feasible solutions over infeasible solutions*: At present, some researchers focus mainly on how to design the search algorithms and the feasibility rule is directly employed or slightly revised to handle constraints. For example, Gordián-Rivera and Mezura-Montes [33] proposed an approach to combine three DE variants, in which the three DE variants compete to generate offspring based on two performance measures. Mezura-Montes and Lopez-Davila [34] designed an adaptive stepsize control and a local search operator, and put them into the modified bacterial foraging algorithm (BFOA) [35]. Hernández-Ocaña *et al.* [36] added four stepsize control mechanisms into the modified BFOA [35]. Elsayed *et al.* carried out a series of work on combining multiple algorithms and/or multiple operators to tackle constrained search space, such as a self-adaptive multi-strategy DE [37] and an adaptive configuration of EAs [38]. Sarker *et al.* [39] developed a DE with dynamic parameter selection. In this method, three sets of parameters are considered: the first set is for the scaling factor *F*, the second is for the crossover control parameter *CR*, and the third is for the population size *NP*. Zhang *et al.* [40] developed a constrained artificial immune system based on immune response principle, in which the population is classified into the feasible and infeasible groups. Tuba and Bacanin [41] hybridized improved seeker optimization algorithm [42] with firefly algorithm [43]. Sadollah *et al.* [44] introduced a new metaheuristic algorithm, called the mine blast algorithm. Mohamed and Sabry [45] implemented several modifications on DE, including the mutation operator, *F*, and *CR*. Recently, Dhadwal *et al.* [46] proposed an advanced particle swarm assisted genetic algorithm.

The ε constrained method proposed by Takahama and Sakai [47] is another representative constraint-handling technique belonging to the methods based on the preference of feasible solutions over infeasible solutions. In 2012, Takahama and Sakai [48] selected different values of *F* and *CR* for each individual in DE according to the rank of the base vector, and proposed a rank-based εDE. In 2013, Takahama and Sakai [49] combined the ε constrained method with the estimated comparison using kernel regression. Recently, Bu *et al.* [50] implemented an improved version of εDEag [51] by utilizing the species based repair strategy. Dominguez-Isidro *et al.* [52] proposed a memetic algorithm, in which DE is used as the global search algorithm and the local search is implemented by a mathematical programming method. In addition, the ε constrained method is applied to compare the individuals.

3) *Methods based on multiobjective optimization*: Currently, this kind of methods usually converts a COP into a biobjective optimization problem like $(f(\vec{x}), G(\vec{x}))$. After the above transformation, Dong and Wang [53] constructed the achievement scalarizing function [54]:

$$F(\vec{x}) = \max\{\omega_1(f(\vec{x}) - z_1), \omega_2(G(\vec{x}) - z_2)\} \quad (7)$$

where $\vec{\omega} = (\omega_1, \omega_2)$ is a weighting vector and $\vec{z} = (z_1, z_2)$ is a reference point. They imposed preference to $f(\vec{x})$ and $G(\vec{x})$ via different weighting vectors and reference points (i.e., a preference based biobjective optimization). Jiao *et al*. [55] proposed a novel selection strategy. After combining the offspring population with the parent population, this selection strategy firstly eliminates the individuals with higher constraint violations than the maximum constraint violation of the last generation, and subsequently the nondominated individuals are chosen. Wang and Cai [56] presented a dynamic hybrid framework referred as DyHF, in which the global and local search models are dynamically implemented according to the feasibility proportion of the population. In the same year, Wang and Cai [57] proposed CMODE, which combines multiobjective optimization with DE. The above two methods adopts Pareto dominance to compare the individuals.

4) *Methods based on hybrid constraint-handling techniques*: Deb and Datta [58] proposed a hybrid evolutionary and penalty function method. This method firstly converts a COP into a biobjective optimization problem. Afterward, NSGA-II [59] is used to solve the converted problem and the nondominated front is applied to estimate the penalty coefficient. In [60], the main framework is inherited from [58], but the previous gradient-based approach is replaced with a gradient-free pattern search approach. Datta and Deb [61] studied on the scaling issue in constrained optimization and proposed an adaptive normalization technique for constraints. Subsequently, this technique is integrated with a hybrid method similar to [58]. Cai *et al*. [62] introduced a novel memetic algorithm called IWO-DE. In IWO-DE, an adaptive fitness function is designed to determine the reproduction ability of each weed in IWO [63]. If the population size of IWO reaches the permissible maximum, some worst individuals are removed by the nondominated sorting [59]. Later, Hu *et al*. [64] extended the above work by making use of a ring neighborhood topology as the population structure. Li and Yin [65] proposed a self-adaptive constrained artificial bee colony algorithm, in which the employed bee colony based on the feasibility rule is responsible for global search and the onlooker bee colony based on multiobjective optimization is treated as the local search model.

IV. Proposed Approach

A. Motivation

Balancing constraints and objective function is a fundamental issue in constrained evolutionary optimization. The methods based on penalty functions attempt to address this issue by introducing appropriate penalty coefficients into the penalty term. In addition, the methods based on multiobjective optimization tend to strike a balance by converting a COP into a multiobjective optimization problem. However, in the methods based on the preference of feasible solutions over infeasible solutions, this issue has not been well studied. For example, despite the feasibility rule being the most popular constraint-handling technique during the last four years, more focus has been put on the search algorithms when using it to solve COPs as pointed out previously. The reason seems straightforward: the feasibility rule prefers constraints to objective function and may cause problems such as premature convergence, especially when solving complex COPs, and as a result, it is expected to design more powerful search algorithms to overcome its limitation to a certain degree. In principle, the feasibility rule is a relatively greedy constrain-handling technique. Note, however, that its greedy property also leads to some attracted advantages over other kinds of constraint-handling techniques, such as the capabilities to rapidly motivate the population toward the feasible region and to speed up the optimization in the promising directions.

```
Input: $NP$: the population size
       $F_{pool}$: the pool of the scaling factor $F$
       $CR_{pool}$: the pool of the crossover control parameter $CR$
       $MaxFEs$: maximum number of fitness evaluations
1:  $t=1$; /* $t$ denotes the generation number */
2:  Randomly generate an initial population $P_t=\{\vec{x}_{1,t},\ldots,\vec{x}_{NP,t}\}$ from the
    decision space $S$;
3:  Evaluate the $f$ value and the $G$ value for each individual in $P_t$;
    /* $f$ and $G$ denote the objective function and the degree of
    constraint violation, respectively */
4:  $FEs=NP$;    /* $FEs$ denotes the number of fitness evaluations */
5:  $P_{t+1}=\emptyset$ and $A=\emptyset$;
6:  For each individual $\vec{x}_{i,t}$ (also called a target vector) in $P_t$
    /* $i=\{1,2,\ldots,NP\}$ */
7:      Randomly select a value from $F_{pool}$ for the scaling factor $F$,
        randomly select a value from $CR_{pool}$ for the crossover control
        parameter $CR$, and implement the mutation and crossover
        operators of DE introduced in Fig. 2 to generate the trial vector $\vec{u}_{i,t}$;
8:      Evaluate the $f$ value and the $G$ value for $\vec{u}_{i,t}$ and set
        $FEs=FEs+1$;
9:      Compare $\vec{x}_{i,t}$ with $\vec{u}_{i,t}$ according to the feasibility rule and store the
        better one into $P_{t+1}$;
10:     If $\vec{u}_{i,t}$ cannot survive into $P_{t+1}$ and if $f(\vec{u}_{i,t})<f(\vec{x}_{i,t})$, then
        $A=A\bigcup\vec{u}_{i,t}$;
11: End For
12: Replace some individuals in $P_{t+1}$ with the individuals in $A$ according
    to the replacement mechanism introduced in Fig. 4;
13: Implement the mutation strategy introduced in Fig. 5 and set
    $FEs=FEs+1$;
14: $t=t+1$;
15: Stopping Criterion: If $FEs\geq MaxFEs$, then stop and output the best
    individual in $P_t$, otherwise go to step 5.
```

Fig. 1. The framework of FROFI

In view of the fast but less reliable constraint-handling performance of the feasibility rule, a new method named FROFI is proposed which utilizes the information provided by objective function to alleviate the greediness and improve the robustness. Moreover, by incorporating the objective function information into the feasibility rule, FROFI can reach an effective balance between constraints and objective function.

B. FROFI

At each generation t, FROFI maintains:

```
If $rand < 0.5$    /* $rand$ is a uniformly distributed
                      random number between 0 and 1 */
    $\vec{v}_{i,t} = \vec{x}_{i,t} + rand \cdot (\vec{x}_{r1,t} - \vec{x}_{i,t}) + F \cdot (\vec{x}_{r2,t} - \vec{x}_{r3,t})$;
    /* DE/current-to-rand/1 */
    $\vec{u}_{i,t} = \vec{v}_{i,t}$;
Else
    $\vec{v}_{i,t} = \vec{x}_{r1,t} + rand \cdot (\vec{x}_{best,t} - \vec{x}_{r1,t}) + F \cdot (\vec{x}_{r2,t} - \vec{x}_{r3,t})$, where $\vec{x}_{best,t}$ is the
    best individual of the population in terms of objective
    function;  /* DE/rand-to-best/1 */
    Execute the binomial crossover of DE on $\vec{x}_{i,t}$ and $\vec{v}_{i,t}$ to
    generate the trial vector $\vec{u}_{i,t}$;
End If
```

Fig. 2. The mutation and crossover operators of DE in FROFI

- a population of NP individuals: $P_t = \{\vec{x}_{1,t}, \ldots, \vec{x}_{NP,t}\}$;
- the objective function values of P_t: $f(\vec{x}_{1,t}), \ldots, f(\vec{x}_{NP,t})$;
- the degree of constraint violation of P_t: $G(\vec{x}_{1,t}), \ldots, G(\vec{x}_{NP,t})$.

COEAs include two main components, i.e., the constraint-handling technique and the search algorithm. Due to its numerous advantages, including simplicity, efficiency, and ease of implementation, DE has been utilized as the search algorithm in FROFI. The framework of FROFI has been given in Fig. 1. During the evolution, for each individual $\vec{x}_{i,t}$ (also called a target vector) in P_t, a trial vector $\vec{u}_{i,t}$ is generated by making use of the mutation and crossover operators of DE. Afterward, $\vec{x}_{i,t}$ is compared with $\vec{u}_{i,t}$ based on the feasibility rule, and the better one is selected and put into the next population P_{t+1}. If $\vec{u}_{i,t}$ cannot survive into P_{t+1} and if $f(\vec{u}_{i,t}) < f(\vec{x}_{i,t})$, then $\vec{u}_{i,t}$ will be stored into a predefined archive A. Under this condition, the properties of $\vec{u}_{i,t}$ can be summarized as follows.

Theorem 1: $\vec{u}_{i,t}$ is an infeasible individual.

Proof: Assume that $\vec{u}_{i,t}$ is a feasible individual. Since $\vec{u}_{i,t}$ is worse than $\vec{x}_{i,t}$ based on the feasibility rule, the following condition holds: $\vec{x}_{i,t}$ is also a feasible individual and $f(\vec{x}_{i,t}) < f(\vec{u}_{i,t})$. This is in contradiction to the fact that $f(\vec{u}_{i,t}) < f(\vec{x}_{i,t})$.

After the update of P_{t+1} has been completed, the individuals in A are used to replace some individuals in P_{t+1} by a replacement mechanism. Subsequently, a mutation strategy is implemented. The above procedure is repeated until the maximum number of fitness evaluations (FEs) is reached.

Next, we will explain the DE operators, the replacement mechanism, and the mutation strategy in detail.

C. DE operators

In FROFI, two DE mutation operators introduced in Section II-A (i.e., DE/current-to-rand/1 and DE/rand-to-best/1) are adopted, each of which is applied with the same probability (i.e., 0.5) when producing the mutant vector $\vec{v}_{i,t}$ for a target vector $\vec{x}_{i,t}$. In DE/current-to-rand/1, the current individual learns the information from other randomly chosen individuals. However, in DE/rand-to-best/1 the information of the best individual in the population is also utilized. In order to further improve the search performance, the first scaling factor in both of them is set to a uniformly distributed random number between 0 and 1. Note that after mutation, the binomial crossover of DE is only applied to DE/rand-to-best/1. DE/current-to-rand/1 without the binomial crossover is a rotation-invariant process and very effective for solving the rotated problems [66]. The details of the DE mutation and crossover operators have been given in Fig. 2.

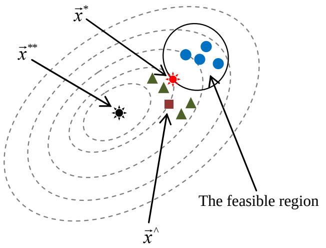


Fig. 3. Schematic diagram of a COP. The dashed ellipses display the contours of objective function, $\vec{x}^{*}$ denotes the optimal solution of this COP which is located on the boundary of the feasible region, $\vec{x}^{**}$ denotes the unconstrained optimal solution, circles denote the feasible individuals, $\vec{x}^{\wedge}$ denotes the infeasible individual with the best objective function value, and triangles denote some potential infeasible individuals near the feasible region which are generated through exploiting $\vec{x}^{\wedge}$.

As shown in Fig. 2, the best individual (i.e., $\vec{x}_{best,t}$) in DE/rand-to-best/1 is determined according to objective function. The reasons are listed as follows:

- At the early stage of evolution, the population may contain only infeasible solutions. Due to that fact that in this scenario the population should continuously approach the feasible region to find a feasible solution, the infeasible individual with the best objective function value may change from generation to generation in the evolutionary process. As a result, $\vec{x}_{best,t}$ somehow likes a randomly selected individual. Under this condition, both DE/rand-to-best/1 and DE/current-to-rand/1 play a similar role, i.e., promoting the global exploration ability of the population.
- At the middle and later stages of evolution, more and more individuals in the population become feasible. If a feasible individual has the best objective function value, then the population will be promptly guided toward this promising solution. On the other hand, if an infeasible individual near the feasible region has the best objective function value, it is very likely that the optimal solution is located on the boundary of the feasible region. In this case, by utilizing such infeasible individual, a lot of potential infeasible individuals may be generated near the feasible region, which provides an advantage to search for the optimal solution by surrounding the boundary of the feasible region from both the feasible and infeasible sides. A COP has been taken as an example in Fig. 3, where $\vec{x}^{*}$ denotes the optimal solution located on the boundary

```
Sort $P_{t+1}$ in descending order according to the objective function values and divide it into $MRN$ parts with the same size;
$i = 1$;
While $|A| > 0$ and $i \le MRN$
    /* $|A|$ denotes the cardinality of $A$ */
    Select the individual with the maximum degree of constraint violation (denoted as $\vec{x}_a$) from the ith part of $P_{t+1}$;
    Select the individual with the minimum degree of constraint violation (denoted as $\vec{x}_b$) from $A$;
    If $f(\vec{x}_b) < f(\vec{x}_a)$
        $P_{t+1} = P_{t+1} \setminus \vec{x}_a$ and $P_{t+1} = P_{t+1} \bigcup \vec{x}_b$;
        $A = A \setminus \vec{x}_b$;
    End If
    $i = i + 1$;
End While
```

Fig. 4. The replacement mechanism in FROFI

of the feasible region and $\vec{x}^{\wedge}$ denotes the infeasible individual with the best objective function value. As shown in Fig. 3, some potential infeasible individuals near the feasible region (denoted as triangles) may be generated via making use of $\vec{x}^{\wedge}$.

Based on the above discussion, it is beneficial to use the information provided by objective function to guide the search in DE. By doing this, the global search ability can be strengthened at the early stage and the local exploitation can be encouraged at the middle and later stages.

D. Replacement Mechanism

The replacement mechanism aims at alleviating the greediness of the feasibility rule by replacing some individuals in the population with the individuals in the archive A.

In order to avoid the replacement occurrence only in a small area of the decision space, a simple way is proposed in which we divide the population into *MRN* parts with the same size, after sorting it based on the objective function values in descending order. This way can be regarded as a very simple and cheap niching technique [67] because the objective function values of the neighboring individuals may be very similar. Subsequently, we choose the individual with the maximum degree of constraint violation from the first part (denoted as $\vec{x}_a$) and the individual with the minimum degree of constraint violation from A (denoted as $\vec{x}_b$), respectively. If $f(\vec{x}_b) < f(\vec{x}_a)$, $\vec{x}_b$ is stored into the population by replacing $\vec{x}_a$ and then deleted from A.

Next, the individual with the maximum constraint violation value in the second part (also denoted as $\vec{x}_a$) and the individual with the minimum constraint violation value in A (also denoted as $\vec{x}_b$) are selected. Similarly, if $f(\vec{x}_b) < f(\vec{x}_a)$, $\vec{x}_a$ is replaced with $\vec{x}_b$ and $\vec{x}_b$ is subsequently removed from A. The above process continues until all the *MRN* parts are updated or A becomes an empty set. Therefore, *MRN* determines the maximum replacement number. Fig. 4 shows the implementation of the replacement mechanism.

The advantages of the replacement mechanism are twofold:

```
If all the individuals in the population are infeasible
    Randomly select an individual (denoted as $\vec{x}_c$) from $P_{t+1}$;
    Generate a random integer number (denoted as $k$) between 1 and $D$, and let the kth dimension of $\vec{x}_c$ be equal to a value randomly chosen from $[L_k, U_k]$. Thus, a mutated individual $\vec{x}_d$ is obtained;
    Evaluate the $f$ value and the $G$ value for $\vec{x}_d$;
    Choose the individual with the maximum degree of constraint violation (denoted as $\vec{x}_e$) in $P_{t+1}$;
    If $f(\vec{x}_d) < f(\vec{x}_e)$
        $P_{t+1} = P_{t+1} \setminus \vec{x}_e$ and $P_{t+1} = P_{t+1} \bigcup \vec{x}_d$;
    End If
End If
```

Fig. 5. The mutation strategy in FROFI

- If the population contains only infeasible individuals, it is helpful to maintain the diversity of the population and guide the population toward the feasible region from diverse directions, by making use of the individuals in A to replace some individuals of the population located in different areas based on objective function.
- For a kind of COPs in which one or several constraints are active at the optimal solution, the optimal solution is located precisely on the boundary of the feasible region. Under this condition, by comparing the individuals based on objective function in the replacement mechanism, the infeasible individuals in the vicinity of the optimal solution are very likely to enter the next population, provided that the objective function values of such infeasible individuals are less than those of the feasible individuals. As pointed out by Mezura-Montes and Coello Coello [8], it is very promising to quickly find the optimal solution located on the boundary of the feasible region by combining the infeasible individuals with the feasible individuals close to the optimal solution.

E. Mutation Strategy

The constraints of some COPs exhibit nonlinear and multimodal properties. As a result, if only the information of constraints is considered, the population will be easily trapped into a local optimum in the infeasible region and feasible solutions cannot be found when the iteration terminates. In this case, the objective function information may be useful for the population to jump out of a local optimal basin in the infeasible region.

Based on the above consideration, a simple mutation strategy is proposed. It is necessary to emphasize that this mutation strategy is only applied to the situation that all the individuals in the population are infeasible. Firstly, let $\vec{x}_c$ be an individual chosen from the population at random, $\vec{x}_e$ the individual with the maximum degree of constraint violation in the population, and k an integer number randomly selected from $[1, D]$. Then, a random number between L_k and U_k is assigned to the *k*th dimension of $\vec{x}_c$ and thus a mutated

TABLE I
THE PRINCIPLES OF THE FEASIBILITY RULE, MULTIOBJECTIVE OPTIMIZATION, AND FROFI WHEN COMPARING A PARENT $\vec{x}_i$ WITH AN OFFSPRING $\vec{u}_i$

Comparison of $\vec{x}_i$ and $\vec{u}_i$	The feasibility rule	Multiobjective optimization	FROFI
$\vec{x}_i$ & $\vec{u}_i$ are infeasible	The importance of the constraint violation is 100% and the importance of the objective function is 0%	The importance of both the constraint violation and objective function is 50%	The constraint violation plays a primary role and the objective function plays an auxiliary role
One is feasible and the other one is infeasible			
$\vec{x}_i$ & $\vec{u}_i$ are feasible	The importance of the objective function is 100% and the importance of the constraint violation is 0%	The importance of the objective function is 100% and the importance of the constraint violation is 0%	The importance of the objective function is 100% and the importance of the constraint violation is 0%

individual is generated (denoted as $\vec{x}_d$). Afterward, if $f(\vec{x}_d) < f(\vec{x}_e)$, $\vec{x}_d$ will enter the population by replacing $\vec{x}_e$. Fig. 5 presents the detailed explanations of the mutation strategy.

Remark: From the above introduction, the implementation of FROFI is simple and it does not impose any computationally expensive operations. The computational time complexity of FROFI is $O(NP\log(NP))$, which is governed by the sorting in the replacement mechanism.

F. *Analysis of the Principle*

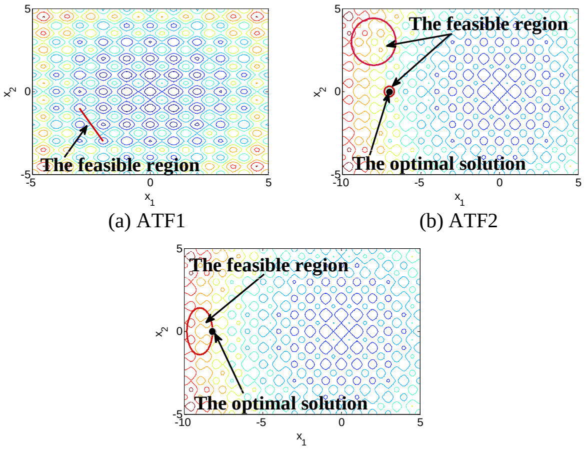
As introduced previously, the feasibility rule and multiobjective optimization are two kinds of popular constraint-handling techniques, and the aim of FROFI is to incorporate the objective function information into the feasibility rule. Indeed, the above three methods have a similarity, i.e., they treat the constraints and objective function separately. Table I discusses the principles of the feasibility rule, multiobjective optimization, and FROFI when comparing a parent $\vec{x}_i$ with an offspring $\vec{u}_i$. Note that Pareto dominance is used to compare $\vec{x}_i$ and $\vec{u}_i$ when a COP is transformed into a multiobjective optimization problem.

As shown in Table I, if both $\vec{x}_i$ and $\vec{u}_i$ are feasible solutions, then all the above three methods put the importance of 100% on the objective function and the constraint violation can be ignored. However, if one of $\vec{x}_i$ and $\vec{u}_i$ is infeasible or both of them are infeasible, the feasibility rule lies on one extreme, i.e., minimizing the constraint violation is considered more important than minimizing the objective function. Meanwhile, multiobjective optimization lies on the other extreme, i.e., the constraint violation and objective function are of equal importance. Under this condition, FROFI first compares $\vec{u}_i$ with $\vec{x}_i$ based on the feasibility rule. Afterward, if $\vec{u}_i$ is worse than $\vec{x}_i$ in terms of the constraint violation and better than $\vec{x}_i$ in terms of the objective function, $\vec{u}_i$ will be stored into the archive and utilized in the subsequent evolution, which means that the constraint violation plays a primary role and the objective function plays an auxiliary role. Therefore, FROFI lies between the above two extremes, which is an advantage of FROFI in principle as compared to the feasibility rule and mutiobjective optimization.

V. EXPERIMENTAL STUDY

A. *Proof-of-Principle Results*

Firstly, three artificial test functions are constructed to capture some important characteristics of introducing the objective function information into the feasibility rule in FROFI. These test functions contain two decision variables, and consequently, are easy to visualize. In order to further compare FROFI with the feasibility rule and multiobjective optimization, two FROFI variants referred as FROFI_FR and FROFI_MO are designed. In FROFI_FR and FROFI_MO, the comparison of individuals is based on the feasibility rule and Pareto dominance, respectively. For both of them, the following two steps ae implemented: 1) the archiving and replacement are removed from FROFI, and 2) the mutation strategy is removed from FROFI. In addition, $\vec{x}_{best,t}$ of DE/rand-to-best/1 in Fig. 2 represents the best individual in the population based on the feasibility rule for FROFI_FR, and one of the nondominated individuals in the population for FROFI_MO. Moreover, DE/current-to-rand/1 in Fig. 2 is kept unchanged for FROFI_FR and FROFI_MO.



(a) ATF1 (b) ATF2

(c) ATF3

Fig. 6. The search space, the contours of objective function, the feasible region, and the optimal solution of the three artificial test functions

The three artificial test functions have the following formulations:

$$\text{ATF1:}\quad \text{minimize}\quad f(\vec{x}) = \sum\nolimits_{i=1}^{2} x_i^2 - 10\cos(2\pi x_i) + 10,\quad -5 \le x_1, x_2 \le 5$$
$$\text{subject to: } x_1 + 2 \le 0$$
$$-x_1 - 3 \le 0$$
$$2x_1 + x_2 + 7 = 0$$

$$\text{ATF2:}\quad \text{minimize}\quad f(\vec{x}) = \sum\nolimits_{i=1}^{2} x_i^2 - 10\cos(2\pi x_i) + 10,$$
$$-10 \le x_1 \le 5,\quad -5 \le x_2 \le 5$$
$$\text{subject to: } 3(x_1+7)^2 + x_2^2 \le 0.3 \text{ or}$$
$$(x_1+8)^2 + (x_2-3)^2 \le 2$$

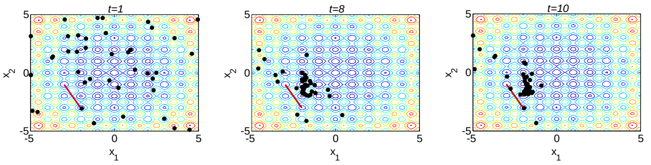


Fig. 7. The evolution of FROFI_FR over a typical run on ATF1. Hereinafter, *t* denotes the generation number.

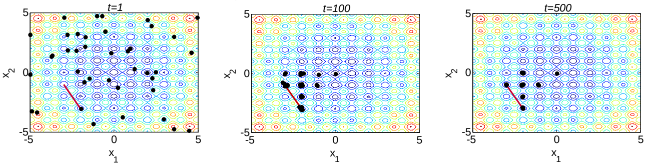


Fig. 8. The evolution of FROFI_MO over a typical run on ATF1

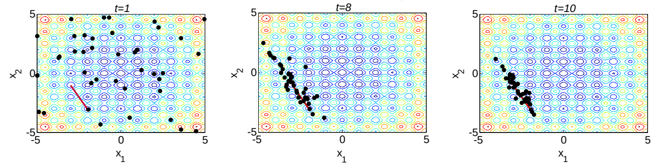


Fig. 9. The evolution of FROFI over a typical run on ATF1

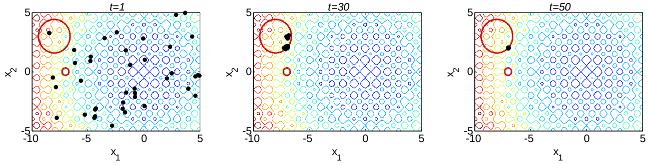


Fig. 10. The evolution of FROFI_FR over a typical run on ATF2

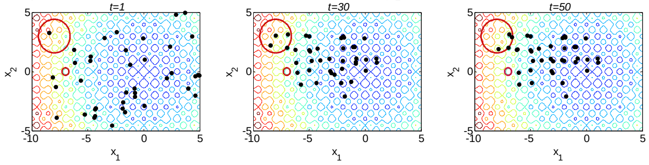


Fig. 11. The evolution of FROFI_MO over a typical run on ATF2

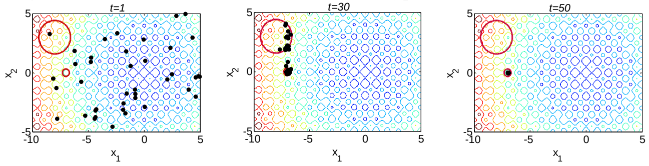


Fig. 12. The evolution of FROFI over a typical run on ATF2

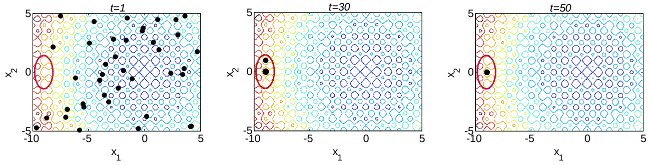


Fig. 13. The evolution of FROFI_FR over a typical run on ATF3

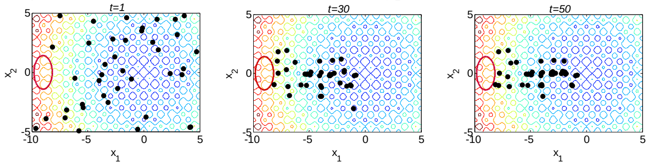


Fig. 14. The evolution of FROFI_MO over a typical run on ATF3

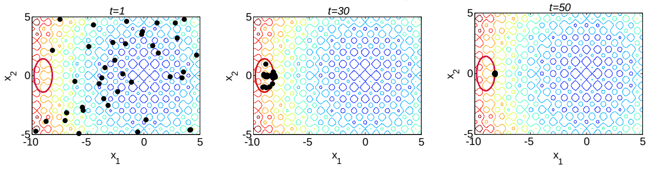


Fig. 15. The evolution of FROFI over a typical run on ATF3

$$\begin{aligned} \text{ATF3:}\quad \text{minimize}\quad & f(\vec{x}) = \sum_{i=1}^{2} x_i^2 - 10\cos(2\pi x_i) + 10, \\ & -10 \le x_1 \le 5,\ \ -5 \le x_2 \le 5 \\ \text{subject to:}\quad & 3(x_1+9)^2 + x_2^2 \le 2 \end{aligned}$$

It is evident that these three test functions have the same objective function, which is the Rastrigin function with 2D [68]. The search space, the contours of objective function, the feasible region, and the optimal solution of them have been presented in Fig. 6.

When solving these three test functions, the population size *NP* was set to 40 and other parameter settings were kept unchanged which will be specified in Section V-B. Moreover, FROFI, FROFI_FR, and FROFI_MO used the same initial population to ensure the comparison fair. Figs. 7-15 provide a typical run derived from them on ATF1, ATF2, and ATF3. From Figs. 7-15, we can observe:

- As depicted in Fig. 7, FROFI_FR approaches the feasible region of ATF1 from only one side along with the evolution. In contrast, FROFI is able to approach the feasible region of ATF1 from both sides as shown in Fig. 9.
- From Fig. 10, FROFI_FR concentrates its search around the feasible area with a relatively larger size of ATF2 and runs the risk of getting stuck at a local feasible optimal solution. On the contrary, the search of FROFI is carried out around the two parts of the feasible region of ATF2 and finally the global optimal solution can be found as shown in Fig. 12.
- It is clear from Fig. 13 that FROFI_FR enters the feasible region of ATF3 with a very fast speed and all the individuals in the population promptly become feasible. Due to the lack of sufficient sampling in the area including the optimal solution, FROFI_FR is prone to converge to a local attraction basin of the feasible region. As shown in Fig. 15, a lot of effort has been made by FROFI on both the feasible and infeasible areas around the optimal solution, and as a result, FROFI succeeds in locating the optimal solution.
- When comparing two infeasible individuals based on Pareto dominance, they may be frequently nondominated with each other. Due to such low selection pressure, it is a very challenging task for FROFI_MO to find feasible solutions, see, for example, Fig. 8, Fig. 11, and Fig. 14. Specifically, for ATF1, FROFI_MO cannot find a feasible solution even the generation number is equal to 500. For ATF2, FROFI_MO fails to find a feasible solution in the small part of the feasible region. In addition, FROFI_MO is unable to provide a feasible solution for ATF3. As pointed out in [12], a search bias toward the feasible region should be introduced into multiobjective optimization for locating feasible solutions of COPs.
- Overall, by incorporating the objective function information into the feasibility rule, FROFI is capable of enhancing the diversity of the population and steering the population toward the feasible region from diverse directions (for instance ATF1 and ATF2). Moreover,

TABLE II
THE MAXIMUM NUMBER OF FEs *MaxFEs* AND THE POPULATION SIZE *NP*

Test Function	*MaxFEs*	*NP*
24 test functions from IEEE CEC2006	5.0E+05	80
18 test functions with 10D from IEEE CEC2010	2.0E+05	60
18 test functions with 30D from IEEE CEC2010	6.0E+05	80

TABLE III
RESULTS OF THE MULTIPLE-PROBLEM WILCOXON'S TEST BASED ON THE SUCCESS PERFORMANCE FOR εDE [47], APF-GA [69], (μ+λ)-CDE [31], DYHF [56], CMODE [57], AND FROFI ON 24 TEST FUNCTIONS FROM IEEE CEC2006

Algorithm	R+	R-	*p*-value	α=0.05	α=0.1
FROFI vs εDE	203.0	73.0	4.84E-02	Yes	Yes
FROFI vs AGF-GA	269.0	7.0	4.53E-06	Yes	Yes
FROFI vs (μ+λ)-CDE	245.0	31.0	5.52E-04	Yes	Yes
FROFI vs DyHF	208.5	67.5	3.14E-02	Yes	Yes
FROFI vs CMODE	245.5	30.5	5.14E-04	Yes	Yes

TABLE IV
RANKING OF εDE [47], APF-GA [69], (μ+λ)-CDE [31], DYHF [56], CMODE [57], AND FROFI BY THE FRIEDMAN'S TEST IN TERMS OF THE SUCCESS PERFORMANCE ON 24 TEST FUNCTIONS FROM IEEE CEC2006

Algorithm	Ranking
FROFI	**2.2708**
εDE	2.75
DyHF	2.8542
CMODE	3.6458
(μ+λ)-CDE	4.0833
AGF-GA	5.3958

it is an effective method to find the optimal solution located on the boundary of the feasible region from both feasible and infeasible parts (for instance ATF3).

To summarize, the above experiments present detailed insights into why FROFI is able to overcome the weaknesses of the two extremes in the feasibility rule and multiobjective optimization. The experiment comparisons among FROFI, FROFI_FR, and FROFI_MO on the benchmark test functions introduced in Section V-B have been summarized in the supplemental file (Tables S1-S3).

B. Benchmark Test Functions and Parameter Settings

Next, we employed two sets of benchmark test functions to thoroughly evaluate the performance of FROFI and to compare FROFI with other state-of-the-art COEAs. The first set is the 24 benchmark test functions collected in IEEE CEC2006 [21], and the second set is the 18 benchmark test functions with 10 dimensions (10D) and 30 dimensions (30D) developed in IEEE CEC2010 [22]. Note that the objective function of all the test functions should be minimized. The details of these test functions can be found in [21] and [22].

In the experimental study of FROFI, the maximum number of FEs *MaxFEs* and the population size *NP* were given in Table II. Note that a proper setting of the population size is related to the dimension of an optimization problem. As shown in Table II, for the 10D test functions, a slightly smaller population size was adopted to make FROFI more efficient. In addition, 25 independent runs were performed for each test function and the tolerance value δ for equality constraints was set to 0.0001. It is necessary to point out that the settings of *MaxFEs*, the number of runs, and the tolerance value δ are based on the suggestions in [21] and [22], and kept the same in all the compared methods. Inspired by [66], we established a scaling factor pool (i.e., $F_{pool} =$ [0.6, 0.8, 1.0]) and a crossover control parameter pool (i.e., $CR_{pool} =$ [0.1, 0.2, 1.0]) in DE. At each generation, we randomly chose a value from F_{pool} for *F* and a value from CR_{pool} for *CR*. Then, the mutation and crossover operators of DE were implemented based on the chosen *F* and *CR* values. FROFI introduces a maximum replacement number *MRN* in the replacement mechanism. In all simulations, *MRN*=max(5,*D*/2) which implies that when *D*>10, *MRN*=*D*/2, otherwise *MRN*=5.

C. Experiments on the 24 Benchmark Test Functions Collected in IEEE CEC2006

For the 24 benchmark test functions (denoted as g01-g24) collected in IEEE CEC2006, the performance of FROFI was compared with that of five state-of-the-art methods: εDE [47], APF-GA [69], (μ+λ)-CDE [31], DyHF [56], and CMODE [57]. The experimental results of these five methods were directly taken from the original papers for fair comparison.

For each test function, a run is successful if the following success condition is satisfied: $f(\vec{x}_{best}) - f(\vec{x}^*) \le 0.0001$ and $\vec{x}_{best}$ is feasible, where $\vec{x}^*$ is the best known solution and $\vec{x}_{best}$ is the best solution provided by an algorithm. Similar to [31], [56], and [57], FROFI finds an improved best known solution for g17, the objective function value of which is 8853.53387481. Therefore, this improved best known solution is used to compute the success condition for g17. Regarding g20, no feasible solution has been reported by the existing algorithms. Moreover, the decrease of constraint violation of an individual in the vicinity of the optimal solution will result in the increase of its objective function value. Thus, for g20 the success condition is revised to $| f(\vec{x}_{best}) - f(\vec{x}^*) | \le 0.0001$.

According to the suggestion in [21], we used the success rate and the success performance as the performance indicators to compare εDE, APF-GA, (μ+λ)-CDE, DyHF, CMODE, and FROFI. The success rate is the percentage of successful runs, and the success performance is the mean number of FEs for successful runs divided by the success rate. The success rates resulting from the six compared methods are summarized in the supplemental file (Table S4). As shown in the supplemental file, εDE, APF-GA, (μ+λ)-CDE, DyHF, CMODE, and FROFI achieve 100% success rate on 22, 12, 21, 22, 22, and 23 test functions, respectively. In this regard, FROFI shows the most stable performance. Moreover, FROFI provides the highest mean success rate (i.e., 95.83%).

The success performance of the six compared methods is also summarized in the supplemental file (Table S5), in

TABLE V
Results of the Multiple-problem Wilcoxon's Test for εDEag [51], SRS-εDEag [50], ECHT-DE [72], AIS-IRP [40], DyHF [56], CMODE [57], and FROFI on 18 Test Functions with 10D from IEEE CEC2010

Algorithm	R+	R-	p-value	α=0.05	α=0.1
FROFI vs εDEag	106.0	65.0	3.193E-01	No	No
FROFI vs SRS-εDEAG	101.0	70.0	4.811E-01	No	No
FROFI vs ECHT-DE	135.0	36.0	3.036E-02	Yes	Yes
FROFI vs AIS-IRP	124.0	47.0	9.874E-02	No	Yes
FROFI vs DyHF	166.5	4.5	6.485E-05	Yes	Yes
FROFI vs CMODE	148.0	5.0	1.526E-04	Yes	Yes

TABLE VI
Ranking of εDEag [51], SRS-εDEag [50], ECHT-DE [72], AIS-IRP [40], DyHF [56], CMODE [57], and FROFI by the Friedman's Test on 18 Test Functions with 10D from IEEE CEC2010

Algorithm	Ranking
FROFI	**2.8611**
SRS-εDEAG	3.0833
εDEag	3.3333
AIS-IRP	3.5278
ECHT-DE	4.3333
CMODE	5.2222
DyHF	5.6389

TABLE VII
Results of the Multiple-problem Wilcoxon's Test for εDEag [51], SRS-εDEag [50], ECHT-DE [72], AIS-IRP [40], DyHF [56], CMODE [57], and FROFI on 18 Test Functions with 30D from IEEE CEC2010

Algorithm	R+	R-	p-value	α=0.05	α=0.1
FROFI vs εDEag	161.5	9.5	2.899E-04	Yes	Yes
FROFI vs SRS-εDEag	133.5	37.5	3.637E-02	Yes	Yes
FROFI vs ECHT-DE	147.5	5.5	1.831E-04	Yes	Yes
FROFI vs AIS-IRP	133.0	20.0	5.57E-03	Yes	Yes
FROFI vs DyHF	153.0	0.0	1.526E-05	Yes	Yes
FROFI vs CMODE	169.5	1.5	1.907E-05	Yes	Yes

TABLE VIII
Ranking of εDEag [51], SRS-εDEag [50], ECHT-DE [72], AIS-IRP [40], DyHF [56], CMODE [57], and FROFI by the Friedman's Test on 18 Test Functions with 30D from IEEE CEC2010

Algorithm	Ranking
FROFI	**1.8611**
SRS-εDEag	2.6944
AIS-IRP	3.6389
εDEag	3.75
ECHT-DE	4.5556
CMODE	5.25
DyHF	6.25

which "*NA*" denotes the success performance of the corresponding method cannot be available since the success rate is equal to zero. To detect the statistical differences systematically, the multiple-problem Wilcoxon's test and the Friedman's test [70] were carried out by making use of keel software [71]. In the Friedman's test, the Bonferroni-Dunn method was chosen for the post-hoc test. Tables III and IV summarize the statistical test results based on the success performance. From Table III, we can observe that FROFI provides higher R+ values than R- values in all the cases. Moreover, the p values of all the cases are less than 0.05, which indicates that FROFI exhibits statistically superior convergence performance against the five competitors. In addition, it can be seen from Table IV that FROFI works best, followed by εDE.

The above comparison verifies that FROFI is better than the five competitors on the 24 benchmark test functions from IEEE CEC2006, in terms of the success rate and the success performance.

D. Experiments on the 18 Benchmark Test Functions with 10D and 30D Designed in IEEE CEC2010

In this subsection, we compared FROFI against six competitive methods on the 18 test functions (denoted as C01-C18) with 10D and 30D from IEEE CEC2010 to validate its performance: εDEag [51], SRS-εDEag [50], ECHT-DE [72], AIS-IRP [40], DyHF [56], and CMODE [57].

Unlike the test functions in Section V-C, the optimal solutions of these 18 test functions cannot be known a priori. Consequently, the average and standard deviation of the objective function values obtained in 25 runs were considered as the performance indicator. The supplemental file (Tables S6-S9) summarizes the experimental results provided by the six compared methods. Herein, "*" denotes that feasible solutions cannot be consistently found by the corresponding method in all runs, and (#) denotes the feasible rate which is the percentage of runs where at least one feasible solution is found when the evolution halts. The experimental results of εDEag, SRS-εDEag, ECHT-DE, and AIS-IRP were directly taken from the original papers. For DyHF and CMODE, we run the source codes provided by the authors in [56] and [57] to produce the experimental results due to the fact that the experimental results cannot be obtained from [56] and [57].

To test the statistical significance, t-test at a 0.05 significance level was conducted between FROFI and each of εDEag, SRS-εDEag, ECHT-DE, and AIS-IRP. In addition, Wilcoxon's rank sum test at a 0.05 significance level is implemented between FROFI and each of DyHF and CMODE. Further, by making use of KEEL software [71], the multiple-problem Wilcoxon's test and the Friedman's test were carried out based on the average objective function values.

In the case of D=10, the supplemental file (Tables S6 and S7) show that FROFI has an edge over εDEag, SRS-εDEag, ECHT-DE, AIS-IRP, DyHF, and CMODE on six, four, 10, nine, 15, and 14 test functions, respectively. In contrast, εDEag, SRS-εDEag, ECHT-DE, AIS-IRP, DyHF, and CMODE perform better than FROFI on four, two, four, five, one, and one test function, respectively. Therefore, we can conclude that, overall, the performance of FROFI is superior to that of the other six competitors.

Table V and Table VI report the statistical test results based on the multiple-problem Wilcoxon's test and the Friedman's test when D=10. As shown in Table V, FROFI provides higher R+ values than R– values in all the cases. In

TABLE IX
RESULTS OF THE MULTIPLE-PROBLEM WILCOXON'S TEST FOR FROFI AND FROFI_WOR ON 18 TEST FUNCTIONS WITH 10D AND 30D FROM IEEE CEC2010

Algorithm	R+	R-	*p*-value	*α*=0.05	*α*=0.1
FROFI vs FROFI_WoR (10D)	145.0	26.0	7.69E-03	Yes	Yes
FROFI vs FROFI_WoR (30D)	159.0	12.0	5.34E-04	Yes	Yes

TABLE X
COMPARISON OF FROFI WITH FROFI_WOM ON C11 WITH 10D, C12 WITH 10D, AND C11 WITH 30D IN TERMS OF THE FEASIBLE RATE

Test Function	Feasible Rate	
	FROFI	FROFI_WoM
C11 with 10D	100%	48%
C12 with 10D	100%	84%
C11 with 30D	100%	16%

terms of the multiple-problem Wilcoxon's test at *α*=0.1, significant difference can be observed in four cases (i.e., FROFI versus ECHT-DE, FROFI versus AIS-IRP, FROFI versus DyHF, and FROFI versus CMODE), which signifies that FROFI performs much better than ECHT-DE, AIS-IRP, DyHF, and CMODE at *α*=0.1. In addition, we can observe from Table VI that FROFI has the best ranking, followed by SRS-*ε*DEag.

In [22], the 18 test functions have been generalized into 30D. Compared with the test functions with 10D, the test functions with 30D have more complex characteristics, which can be used to test the scalability of a COEA.

In the case of *D*=30, as shown in the supplemental file (Tables S8 and S9), FROFI is remarkably better than the six competitors on a vast majority of test functions. More specifically, FROFI beats *ε*DEag, SRS-*ε*DEag, ECHT-DE, AIS-IRP, DyHF, and CMODE on 14, 10, 14, 14, 17, and 16 test functions, respectively. Nevertheless, *ε*DEag, SRS-*ε*DEag, ECHT-DE, and AIS-IRP outperform FROFI only on two, two, one, and three test functions, respectively. Moreover, DyHF and CMODE cannot surpass FROFI on any test functions.

Table VII and Table VIII summarize the statistical test results based on the multiple-problem Wilcoxon's test and the Friedman's test when *D*=30. From Table VII, it is obvious that FROFI provides higher R+ values than R– values in all the cases. Furthermore, the *p* values of all the cases are less than 0.05, which means that FROFI significantly outperforms the other six competitors. In addition, it can be observed from Table VIII that FROFI has the best ranking, followed by SRS-*ε*DEag.

The above experimental results reveal that FROFI has the increasing advantage over the other compared methods for complex high-dimensional COPs, which also implies that FROFI could be more effective for solving large-scale COPs.

E. Discussion

In this subsection, additional experiments were carried out on the 18 benchmark test functions with 10D and 30D from IEEE CEC2010. For all the experiments, 25 independent runs were executed and all the parameter settings were kept unchanged unless otherwise specified.

TABLE XI
RESULTS OF THE MULTIPLE-PROBLEM WILCOXON'S TEST FOR FROFI_13, FROFI_14, FROFI_15, FROFI_16, AND FROFI_17 ON 18 TEST FUNCTIONS WITH 30D FROM IEEE CEC2010

Algorithm	R+	R-	*p*-value	*α*=0.05	*α*=0.1
FROFI_15 vs FROFI_13	132.0	21.0	8.03E-03	Yes	Yes
FROFI_15 vs FROFI_14	100.5	62.5	3.42E-01	No	No
FROFI_15 vs FROFI_16	86.5	84.5	1.00E+00	No	No
FROFI_15 vs FROFI_17	98.5	72.5	5.57E-01	No	No

TABLE XII
RANKING OF FROFI_13, FROFI_14, FROFI_15, FROFI_16, AND FROFI_17 BY THE FRIEDMAN'S TEST ON 18 TEST FUNCTIONS WITH 30D FROM IEEE CEC2010

Algorithm	Ranking
FROFI_15	**2.6944**
FROFI_16	2.7778
FROFI_17	2.8883
FROFI_14	2.9644
FROFI_13	3.5

1) *Effectiveness of the replacement mechanism:* In FROFI, the external archive stores some infeasible solutions carrying valuable information of objective function. Moreover, such infeasible solutions have been reemployed through the replacement mechanism. We implemented a variant of FROFI, called FROFI_WoR, in which the replacement mechanism has been discarded. The average and standard deviation of the objective function values obtained from FROFI and FROFI_WoR have been given in the supplemental file (Tables S10-S11). Table IX reports the statistical test results according to the multiple-problem Wilcoxon's test.

As shown in the supplemental file (Tables S10-S11), when *D*=10 and 30, FROFI performs better than FROFI_WoR on 12 and 15 test functions, respectively. However, FROFI_WoR outperforms FROFI only on two and one test function, respectively. Moreover, we can observe from Table IX that FROFI provides higher R+ values than R– values in all the cases and the *p* values of all the cases are less than 0.05.

Based on the above comparison, one can conclude that the replacement mechanism does play a crucial role in FROFI.

2) *Effectiveness of the mutation strategy:* We also considered another variant of FROFI, called FROFI_WoM, in which the mutation strategy has been removed. Table X summarizes the experimental results of FROFI and FROFI_WoM for C11 with 10D, C12 with 10D, and C11 with 30D, which means that FROFI and FROFI_WoM achieved quite similar performance on the remaining test functions. After a careful observation, we found that both C11 and C12 use the Ronsenbrock function [68] as the constraint. The global minimum of the Ronsenbrock function is inside a long,

TABLE XIII
EXPERIMENTAL RESULTS OF ABC, TLBO, AND FROFI OVER 100 INDEPENDENT RUNS ON THREE CONSTRAINED MECHANICAL DESIGN OPTIMIZATION PROBLEMS

Problem	The maximum number of FEs	Criteria	ABC	TLBO	FROFI
Step-cone pulley	15000	Best	16.634655	16.634510	14.467584
		Mean	36.099500	24.011358	**14.467699**
		Worst	145.470500	74.022951	14.468038
Hydrostatic thrust bearing	25000	Best	1625.442760	1625.443000	1625.449568
		Mean	1861.554000	1797.707980	**1663.562923**
		Worst	2144.836000	2096.801270	1869.449075
Rolling element bearing	10000	Best	-81859.741600	-81859.740000	-81859.198042
		Mean	-81496.000000	-81438.987000	**-81856.171959**
		Worst	-78897.810000	-80807.855100	-81848.523796

narrow, and parabolic shaped flat valley, and as a result, it is very difficult to find the global minimum. Due to the above property, when using the Ronsenbrock function as the constraint, it is not strange that some methods tend to converge to a local optimum in the infeasible region. For example, εDEag, SRS-εDEag, ECHT-DE, DyHF, and CMODE are incapable of consistently finding feasible solutions on C11 and C12 as shown in the supplemental file (Tables S6-S9).

In Table X, the feasible rate was taken as the performance indicator. From Table X, the feasible rate drastically decreases when implementing FROFI_WoM on C11 with 10D, C12 with 10D, and C11 with 30D. One may be interested in why FROFI and FROFI_WoM show the similar performance on C12 with 30D. This is not difficult to understand because the relatively bigger FEs (i.e., 6×10^5 FEs) has been specified.

The above comparison corroborates that the use of the objective function information in the mutation strategy is helpful for FROFI to avoid premature convergence in the complex constrained search space.

3) *Sensitivity in relation to the parameter associate with the replacement mechanism*: FROFI introduces its own parameter (i.e., *MRN*) which determines the maximum replacement number in the replacement mechanism. To study how the performance of FROFI is sensitive to this parameter, we have tried different values of *MRN*. The performance analysis was performed via the multiple-problem Wilcoxon's test and the Friedman's test based on the mean objective function value.

According to our observation, FROFI is not sensitive to *MRN* on the 18 test functions with 10D in IEEE CEC2010, and *MRN* can be set to a value in a large range. Therefore, we only reported the statistical test results for the 18 test functions with 30D from IEEE CEC2010 in Tables XI and XII. For these test functions, we tested five different values of *MRN*: 13, 14, 15, 16, and 17. FROFI with the above five values are denoted as FROFI_13, FROFI_14, FROFI_15, FROFI_16, and FROFI_17, respectively. Note that FROFI_15 is equivalent to the original FROFI. The average and standard deviation of the objective function values resulting from the compared methods are reported in the supplemental file (Table S12).

From Table XI, FROFI_13 suffers from performance degradation since the *p* value is less than 0.05 when comparing FROFI_13 with FROFI_15. Moreover, FROFI_13 gets the worst ranking as shown in Table XII. On the other hand, it seems that FROFI_14, FROFI_15, FROFI_16, and FROFI_17 have similar overall performance. Thus, we could claim that FROFI is not very sensitive to the setting of *MRN* for the 18 test functions with 30D in IEEE CEC2010.

The above discussion demonstrates that *MRN* is a problem insensitive parameter in FROFI.

F. *FROFI for Constrained Mechanical Design Optimization Problems*

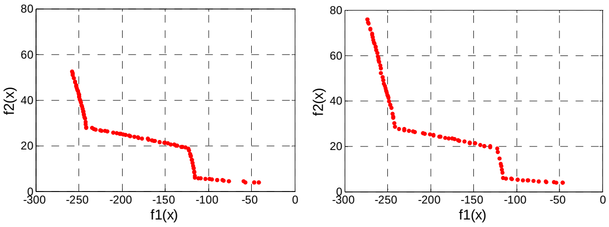
In the previous subsections, the performance of FROFI has been assessed by benchmark test functions. One may be interested in the performance of FROFI in practical applications. To this end, three constrained mechanical design optimization problems introduced in [73] are adopted. We used the same maximum number of FEs as in [73] for these three optimization problems.

Table XIII summarizes the experimental results of ABC, TLBO, and FROFI. Note that the experimental results of ABC and TLBO were directly taken from [73] for fair comparison. From Table XIII, it can be observed that FROFI provides better average results than the two competitors on these three optimization problems, which verifies the effectiveness of FROFI in the practical applications.

G. *Is Our Idea Applicable to Constrained Multiobjective Optimization Problems (CMOPs)?*

In [59], NSGA-II has been integrated with a constrained-domination rule which is an extension of the feasibility rule [7] for solving CMOPs. In NSGA-II, the parent population P_t and the offspring population Q_t are sorted based on the constrained-domination rule, and then all the individuals in $P_t \cup Q_t$ are partitioned into several nondominated levels. Finally, the individuals in $P_t \cup Q_t$ are put into the next population P_{t+1} level by level.

In this paper, according to the characteristics of CMOPs, the objective function information has been incorporated into NSGA-II as follows: 1) the individuals in Q_t are resorting based only on the objective functions and divided into several nondominated levels, 2) if some individuals in the best nondominated level have not been put into P_{t+1}, then they are stored into an archive, and 3) the individual with the minimum constraint violation in the archive (denoted as $\vec{x}_a$) is used to replace the individual with the maximum constraint violation in P_{t+1} (denoted as $\vec{x}_b$) if $\vec{x}_a$ Pareto dominates $\vec{x}_b$. The improved NSGA-II is called INSGA-II in this paper.



(a) NSGA-II (b) INSGA-II

Fig. 16. The Pareto front provided by NSGA-II and INSGA-II in a typical run on OSY.

Due to the limit of the paper length, only the OSY problem in [74] is used to test the performance of NSGA-II and INSGA-II. In addition, the hypervolume (HV) [75] is considered as the performance metric. Note that the larger the HV value, the better the performance of an algorithm. From the experiments, the mean HV values obtained by NSGA-II and INSGA-II over 25 runs are 696.3069 and 712.9837, which suggests that the objective function information can also be applied to enhance the performance of NSGA-II for solving CMOPs.

Fig. 16 exhibits the Pareto fronts resulting from NSGA-II and INSGA-II in a typical run. As shown in Fig. 16, NSGA-II is very likely to miss some parts of the true Pareto front.

VI. Conclusion

This paper proposes an alternative method to balance constraints and objective function in constrained evolutionary optimization, called FROFI. In FROFI, we utilize the information of objective function to alleviate the greediness and improve the robustness of the well-known feasibility rule by three processes, i.e., the DE operators, the replacement mechanism, and the mutation strategy. Moreover, the comparison of individuals is based on objective function in the replacement mechanism and the mutation strategy.

Experiments across two benchmark test sets from IEEE CEC2006 and IEEE CEC2010 show that: 1) the DE operators have the capability to balance the exploration and exploitation during the evolution, 2) the replacement mechanism increases the diversity of the infeasible population, and efficiently searches the optimal solution from both the feasible and infeasible areas when the population contains feasible solutions, 3) the mutation strategy is a promising way to deal with complicated constrained search space, and 4) FROFI achieves better or at least highly competitive performance against other state-of-the-art COEAs. Moreover, the performance advantage of FROFI is more pronounced on high-dimensional test functions. In the future, it is interesting to design adaptive or self-adaptive replacement mechanism in FROFI for solving large-scale COPs.

The Matlab source code of FROFI can be downloaded from Y. Wang’s homepage: http://ist.csu.edu.cn/YongWang.htm

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Yong Wang (M'08) received the B.S. degree in automation from the Wuhan Institute of Technology, Wuhan, China, in 2003, and the M.S. degree in pattern recognition and intelligent systems and the Ph.D. degree in control science and engineering both from the Central South University (CSU), Changsha, China, in 2006 and 2011, respectively.

He is currently an Associate Professor with the School of Information Science and Engineering, CSU. His current research interests include evolutionary computation, single-objective optimization, constrained optimization, multiobjective optimization, and their real-world applications.

Dr. Wang is a member of the IEEE CIS Task Force on Nature-Inspired Constrained Optimization and the IEEE CIS Task Force on Differential Evolution. He was a reviewer of 50+ international journals and a PC member of 30+ international conferences. He was awarded the Hong Kong Scholar by the Mainland - Hong Kong Joint Postdoctoral Fellows Program, China, in 2013, the Excellent Doctoral Dissertation by Hunan Province, China, in 2013, the New Century Excellent Talents in University by the Ministry of Education, China, in 2013, and the 2015 IEEE Computational Intelligence Society Outstanding PhD Dissertation Award. He is currently serving as an associate editor for the Swarm and Evolutionary Computation.

Bing-Chuan Wang was born in Jiangxi, China, in 1990. He received the B.S. degree in automation from Central South University, Changsha, China, in 2013. He is currently working toward the M.S. degree from Central South University, Changsha, China. His current research interests include constrained evolutionary optimization, multiobjective evolutionary optimization, and machine learning.

Han-Xiong Li (S'94-M'97-SM'00-F'11) received his B.E. degree in aerospace engineering from the National University of Defense Technology, China in 1982, M.E. degree in electrical engineering from Delft University of Technology, The Netherlands in 1991, and Ph.D. degree in electrical engineering from the University of Auckland, New Zealand in 1997.

He is a professor in the Department of Systems Engineering and Engineering Management, the City University of Hong Kong. Before that, he has worked in different fields, including military service, banking investment, and semiconductor industry. He authored 2 books and published more than 160 SCI journal papers with h-index 31. His current research interests are in system intelligence and control, process design and control integration, distributed parameter systems with applications to electronics packaging.

Dr. Li serves as Associate Editor of IEEE Transactions on Cybernetics, and IEEE Transactions on Industrial Electronics (2009-2015). He was awarded the Distinguished Young Scholar (overseas) by the China National Science Foundation in 2004, a Chang Jiang professorship by the Ministry of Education, China in 2006, and a national professorship in China Thousand Talents Program in 2010. He serves as a distinguished expert for Hunan Government and China Federation of Returned Overseas. He is a Fellow of the IEEE.

Gary G. Yen (S'87-M'88-SM'97-F'09) received the Ph.D. degree in electrical and computer engineering from the University of Notre Dame in 1992. Currently he is a Regents Professor in the School of Electrical and Computer Engineering, Oklahoma State University (OSU). Before joined OSU in 1997, he was with the Structure Control Division, U.S. Air Force Research Laboratory in Albuquerque. His research is supported by the DoD, DoE, EPA, NASA, NSF, and Process Industry. His research interest includes intelligent control, computational intelligence, conditional health monitoring, signal processing and their industrial/defense applications. He is an IEEE Fellow-class of 2009.

Dr. Yen was an associate editor of the IEEE Control Systems Magazine, IEEE Transactions on Control Systems Technology, Automatica, Mechantronics, IEEE Transactions on Systems, Man and Cybernetics, Parts A and B and IEEE Transactions on Neural Networks. He is currently serving as an associate editor for the IEEE Transactions on Evolutionary Computation and the IEEE Transactions on Cybernetics. He served as the General Chair for the 2003 IEEE International Symposium on Intelligent Control held in Houston, TX and 2006 IEEE World Congress on Computational Intelligence held in Vancouver, Canada. Dr. Yen served as Vice President for the Technical Activities in 2005-2006 and then President in 2010-2011 of the IEEE Computational intelligence Society. He was the founding editor-in-chief of the IEEE Computational Intelligence Magazine, 2006-2009. In 2011, he received Andrew P Sage Best Transactions Paper award from IEEE Systems, Man and Cybernetics Society and in 2014, he received Meritorious Service award from IEEE Computational Intelligence Society.

Supplemental file

Table Captions

- **Table S1** EXPERIMENTAL RESULTS OF FROFI_FR, FROFI_MO, AND FROFI OVER 25 INDEPENDENT RUNS ON 24 TEST FUNCTIONS FROM IEEE CEC2006 USING 5×10^5 FES. "MEAN OFV" AND "STD DEV" INDICATE THE AVERAGE AND STANDARD DEVIATION OF THE OBJECTIVE FUNCTION VALUES OBTAINED IN 25 RUNS, RESPECTIVELY. WILCOXON'S RANK SUM TEST AT A 0.05 SIGNIFICANCE LEVEL WAS PERFORMED BETWEEN FROFI AND EACH OF FROFI_FR AND FROFI_MO.
- **TABLE S2** EXPERIMENTAL RESULTS OF FROFI_FR, FROFI_MO, AND FROFI OVER 25 INDEPENDENT RUNS ON 18 TEST FUNCTIONS WITH 10D FROM IEEE CEC2010 USING 2×10^5 FES. "MEAN OFV" AND "STD DEV" INDICATE THE AVERAGE AND STANDARD DEVIATION OF THE OBJECTIVE FUNCTION VALUES OBTAINED IN 25 RUNS, RESPECTIVELY. WILCOXON'S RANK SUM TEST AT A 0.05 SIGNIFICANCE LEVEL WAS PERFORMED BETWEEN FROFI AND EACH OF FROFI_FR AND FROFI_MO.
- **Table S3** EXPERIMENTAL RESULTS OF FROFI_FR, FROFI_MO, AND FROFI OVER 25 INDEPENDENT RUNS ON 18 TEST FUNCTIONS WITH 30D FROM IEEE CEC2010 USING 6×10^5 FES. "MEAN OFV" AND "STD DEV" INDICATE THE AVERAGE AND STANDARD DEVIATION OF THE OBJECTIVE FUNCTION VALUES OBTAINED IN 25 RUNS, RESPECTIVELY. WILCOXON'S RANK SUM TEST AT A 0.05 SIGNIFICANCE LEVEL WAS PERFORMED BETWEEN FROFI AND EACH OF FROFI_FR AND FROFI_MO.
- **Table S4** COMPARISON OF FROFI WITH RESPECT TO εDE [47], APF-GA [69], (μ+λ)-CDE [31], DYHF [56], AND CMODE [57] IN TERMS OF THE SUCCESS RATE. IN ALL THE EXPERIMENTS, 25 INDEPENDENT RUNS WERE IMPLEMENTED ON 24 TEST FUNCTIONS FROM IEEE CEC2006 USING 5×10^5 FES.
- **Table S5** COMPARISON OF FROFI WITH RESPECT TO εDE [47], APF-GA [69], (μ+λ)-CDE [31], DYHF [56], AND CMODE [57] IN TERMS OF THE SUCCESS PERFORMANCE. IN ALL THE EXPERIMENTS, 25 INDEPENDENT RUNS WERE IMPLEMENTED ON 24 TEST FUNCTIONS FROM IEEE CEC2006 USING 5×10^5 FES.
- **Table S6** EXPERIMENTAL RESULTS OF εDEAG [51], SRS-εDEAG [50], ECHT-DE [72], AIS-IRP [40], AND FROFI OVER 25 INDEPENDENT RUNS ON 18 TEST FUNCTIONS WITH 10D FROM IEEE CEC2010 USING 2×10^5 FES. "MEAN OFV" AND "STD DEV" INDICATE THE AVERAGE AND STANDARD DEVIATION OF THE OBJECTIVE FUNCTION VALUES OBTAINED IN 25 RUNS, RESPECTIVELY. t-TEST AT A 0.05 SIGNIFICANCE LEVEL WAS PERFORMED BETWEEN FROFI AND EACH OF εDEAG, SRS-εDEAG, ECHT-DE, AND AIS-IRP.
- **Table S7** EXPERIMENTAL RESULTS OF DYHF [56], CMODE [57], AND FROFI OVER 25 INDEPENDENT RUNS ON 18 TEST FUNCTIONS WITH 10D FROM IEEE CEC2010 USING 2×10^5 FES. "MEAN OFV" AND "STD DEV" INDICATE THE AVERAGE AND

Standard Deviation of the Objective Function Values Obtained in 25 Runs, Respectively. Wilcoxon's Rank Sum Test at a 0.05 Significance Level was Performed between FROFI and Each of DyHF and CMODE.

- **Table S8** Experimental Results of *ε*DEag [51], SRS-*ε*DEag [50], ECHT-DE [72], AIS-IRP [40], and FROFI over 25 Independent Runs on 18 Test Functions with 30D from IEEE CEC2010 Using 6×10^5 FEs. "Mean OFV" and "Std Dev" Indicate the Average and Standard Deviation of the Objective Function Values Obtained in 25 Runs, Respectively. *t* -Test at a 0.05 Significance Level was Performed between FROFI and Each of *ε*DEag, SRS-*ε*DEag, ECHT-DE, and AIS-IRP.
- **Table S9** Experimental Results of DyHF [56], CMODE [57], and FROFI over 25 Independent Runs on 18 Test Functions with 30D from IEEE CEC2010 Using 6×10^5 FEs. "Mean OFV" and "Std Dev" Indicate the Average and Standard Deviation of the Objective Function Values Obtained in 25 Runs, Respectively. Wilcoxon's Rank Sum Test at a 0.05 Significance Level was Performed between FROFI and Each of DyHF and CMODE.
- **Table S10** Experimental Results of FROFI_WoR and FROFI over 25 Independent Runs on 18 Test Functions with 10D from IEEE CEC2010 Using 2×10^5 FEs. "Mean OFV" and "Std Dev" Indicate the Average and Standard Deviation of the Objective Function Values Obtained in 25 Runs, Respectively. Wilcoxon's Rank Sum Test at a 0.05 Significance Level was Performed between FROFI and FROFI_WoR.
- **Table S11** Experimental Results of FROFI_WoR and FROFI over 25 Independent Runs on 18 Test Functions with 30D from IEEE CEC2010 Using 6×10^5 FEs. "Mean OFV" and "Std Dev" Indicate the Average and Standard Deviation of the Objective Function Values Obtained in 25 Runs, Respectively. Wilcoxon's Rank Sum Test at a 0.05 Significance Level was Performed between FROFI and FROFI_WoR.
- **Table S12** Experimental Results of FROFI_13, FROFI_14, FROFI_15, FROFI_16, and FROFI_17 over 25 Independent Runs on 18 Test Functions with 30D from IEEE CEC2010 Using 6×10^5 FEs. "Mean OFV" and "Std Dev" Indicate the Average and Standard Deviation of the Objective Function Values Obtained in 25 Runs, Respectively.

TABLE S1
EXPERIMENTAL RESULTS OF FROFI_FR, FROFI_MO, AND FROFI OVER 25 INDEPENDENT RUNS ON 24 TEST FUNCTIONS FROM IEEE CEC2006 USING 5×10^5 FES. "MEAN OFV" AND "STD DEV" INDICATE THE AVERAGE AND STANDARD DEVIATION OF THE OBJECTIVE FUNCTION VALUES OBTAINED IN 25 RUNS, RESPECTIVELY. WILCOXON'S RANK SUM TEST AT A 0.05 SIGNIFICANCE LEVEL WAS PERFORMED BETWEEN FROFI AND EACH OF FROFI_FR AND FROFI_MO.

Test Function from IEEE CEC2006	FROFI_FR	FROFI_MO	FROFI
	Mean OFV±Std Dev	Mean OFV±Std Dev	Mean OFV±Std Dev
g01	-15.0000E+01±0.00E+00≈	-6.3245E+01±1.00E+01－**(0%)***	-15.0000E+01±0.00E+00
g02	-8.0323E-01±2.21E-03－	-8.0283E-01±2.75E-03－	-8.0362E-01±1.78E-07
g03	-5.1685E-01±1.04E-01－	-8.3141E-01±3.58E-01－**(56%)***	-1.0005E+00±4.49E-16
g04	-3.066553E+04±3.71E-12≈	-3.066553E+04±3.71E-12≈	-3.066553E+04±3.71E-12
g05	5.1264967E+03±2.63E-05≈	4.756954E+03±4.75E+02－**(0%)***	5.1264967E+03±2.78E-12
g06	-6.9618138E+03±0.00E+00≈	-6.5993020E+03±1.57E+00－	-6.961813E+03±0.00E+00
g07	2.430621E+01±8.14E-15≈	2.431041E+01±9.312E-03－	2.430621E+01±6.32E-15
g08	-9.5825E+02±1.42E-17≈	-9.5825E+02±7.68E-07≈	-9.5825E+02±1.42E-17
g09	6.8063006E+02±2.49E-13≈	6.8063006E+02±3.63E-13≈	6.8063006E+02±3.64E-13
g10	7.0492480E+03±1.99E-12≈	6.5800367E+03±1.41E+03－**(8%)***	7.0492480E+03±3.26E-12
g11	7.499E-01±5.53E-05≈	7.504E-01±5.96E-04－	7.499E-01±1.13E-16
g12	-1.00E+00±0.00E+00≈	-1.00E+00±0.00E+00≈	-1.00E+00±0.00E+00
g13	8.1063E-01±1.00E-01－**(96%)***	1.0399E-01±9.88E-02－**(0%)***	5.3942E-02±2.41E-17
g14	-4.776489E+01±2.90E-14≈	-6.067545E+02±6.41E+01－**(0%)***	-4.776489E+01±2.34E-14
g15	9.6171502E+02±5.80E-13≈	9.6119367E+02±1.90E+00－**(0%)***	9.617150E+02±5.80E-13
g16	-1.90516E+00±4.53E-16≈	-1.89874E+00±8.18E-03－**(96%)***	-1.90516E+00±4.53E-16
g17	8.8774789E+03±3.55E+01－	8.4253151E+03±3.70E+02－**(0%)***	8.853533E+03±0.00E+00
g18	-8.660254E-01±4.79E-15≈	-1.181442E+01±3.89E+00－**(0%)***	-8.66025E-01±6.94E-16
g19	3.265559E+01±2.12E-14≈	3.265559E+01±2.18E-14≈	3.265559E+01±2.18E-14
g20	2.048E-01±1.82E-04≈**(0%)***	6.051E-02±5.22E-03－**(0%)***	2.049E-01±5.31E-05**(0%)***
g21	1.9372451E+02±1.54E-11≈	1.0332682E+02±5.18E+01－**(0%)***	1.937245E+02±2.95E-11
g22	—	—	—
g23	-4.000551E+02±1.28E-13≈	-1.594065E+03±2.41E+02－**(0%)***	-4.000551E+02±1.71E-13
g24	-5.50801E+00±9.06E-16≈	-5.50801E+00±9.06E-16≈	-5.50801E+00±9.06E-16
－	4	17	
＋	0	0	
≈	19	6	

"－", "＋", and "≈" denote that the performance of the corresponding method is worse than, better than, and similar to that of FROFI, respectively. "*" denotes that feasible solutions cannot be consistently found by the corresponding method in all runs, and (#) denotes the feasible rate.

TABLE S2

Experimental Results of FROFI_FR, FROFI_MO, and FROFI over 25 Independent Runs on 18 Test Functions with 10D from IEEE CEC2010 Using 2×10^5 FEs. "Mean OFV" and "Std Dev" Indicate the Average and Standard Deviation of the Objective Function Values Obtained in 25 Runs, Respectively. Wilcoxon's Rank Sum Test at a 0.05 Significance Level was Performed between FROFI and Each of FROFI_FR and FROFI_MO.

Test Function with 10D from IEEE CEC2010	FROFI_FR	FROFI_MO	FROFI
	Mean OFV±Std Dev	Mean OFV±Std Dev	Mean OFV±Std Dev
C01	-7.47E-01±1.4E-13≈	-7.47E-01±1.4E-03≈	-7.47E-01±1.35E-03
C02	2.26E+00±1.19E-01－**(88%)***	1.10E+00±1.50E+00－**(68%)***	-2.02E+00±1.41E-01
C03	0.00E+00±0.00E+00≈	8.52E+00±1.8E+00－	0.00E+00±0.00E+00
C04	-1.00E-05±0.00E+00≈	-2.97E-01±3.31E-01－**(0%)***	-1.00E-05±0.00E+00
C05	3.85E+02±1.48E+02－**(0%)***	2.25E+02±1.10E+02－**(0%)***	-4.84E+02±8.09E-07
C06	3.20E+02±1.92E+02－**(0%)***	7.79E+01±1.80E+02－**(0%)***	-5.79E+02±5.04E-04
C07	0.00E+00±0.00E+00≈	0.00E+00±0.00E+00≈	0.00E+00±0.00E+00
C08	7.84E+00±4.98E+00≈	7.34E+00±5.14E+00≈	7.11E+00±4.79E+00
C09	8.59E+12±4.64E+12－**(0%)***	5.44E+11±6.92E+11－**(0%)***	2.50E+01±3.92E+01
C10	9.65E+12±6.63E+12－**(0%)***	1.29E+12±2.66E+12－**(0%)***	4.17E+01±8.69E-06
C11	-1.52E-03±2.00E-18≈	-1.94E+01±4.38E+00－**(0%)***	-1.52E-03±5.63E-14
C12	-8.23E+13±1.52E+02－**(60%)***	-1.82E+03±2.86E+02－**(0%)***	-3.84E+02±2.17E+02
C13	-6.84E+01±2.90E-09≈	-5.81E+01±9.06E+00－**(36%)***	-6.84E+01±2.52E-09
C14	9.45E+10±1.18E+11－	1.85E+00±5.46E+00－	0.00E+00±0.00E+00
C15	3.12E+13±1.72E+13－	3.71E+11±4.29E+11－	3.09E+00±1.37E+00
C16	1.05E+00±3.03E-02－**(64%)***	7.61E-01±2.66E-01－**(28%)***	1.19E-02±2.07E-02
C17	5.07E+02±2.42E+02－**(52%)***	8.44E+01±7.18E+01－**(40%)***	7.83E-02±2.25E-01
C18	1.01E+04±4.79E+03－**(92%)***	2.59E+03±1.98E+03－**(44%)***	5.23E-26±1.71E-25
－	11	15	
＋	0	0	
≈	7	3	

"－", "＋", and "≈" denote that the performance of the corresponding method is worse than, better than, and similar to that of FROFI, respectively. "*" denotes that feasible solutions cannot be consistently found by the corresponding method in all runs, and (#) denotes the feasible rate.

TABLE S3

EXPERIMENTAL RESULTS OF FROFI_FR, FROFI_MO, AND FROFI OVER 25 INDEPENDENT RUNS ON 18 TEST FUNCTIONS WITH 30D FROM IEEE CEC2010 USING 6×10^5 FES. "MEAN OFV" AND "STD DEV" INDICATE THE AVERAGE AND STANDARD DEVIATION OF THE OBJECTIVE FUNCTION VALUES OBTAINED IN 25 RUNS, RESPECTIVELY. WILCOXON'S RANK SUM TEST AT A 0.05 SIGNIFICANCE LEVEL WAS PERFORMED BETWEEN FROFI AND EACH OF FROFI_FR AND FROFI_MO.

Test Function with 30D from IEEE CEC2010	FROFI_FR	FROFI_MO	FROFI
	Mean OFV±Std Dev	Mean OFV±Std Dev	Mean OFV±Std Dev
C01	-8.21E-01±1.5E-03≈	-8.21E-01±1.69E-03≈	-8.21E-01±2.36E-03
C02	3.40E+00±8.15E-01－	2.65E+00±6.07E-01－	-2.00E+00±4.35E-02
C03	1.89E+12±7.45E+12－	2.87E+01±3.02E-04－**(96%)***	2.87E+01±6.24E-08
C04	2.60E-02±1.30E-02－	-6.57E-01±1.11E-01－**(0%)***	-3.33E-06±4.13E-10
C05	5.30E+02±5.09E+01－**(0%)***	4.10E+02±8.51E+01－**(0%)***	-4.81E+02±2.84E+00
C06	5.57E+02±4.25E+01－**(0%)***	4.25E+02±7.66E+01－**(0%)***	-5.29E+02±5.71E-01
C07	1.60E+00±7.97E-01－	0.00E+00±0.00E+00≈	0.00E+00±0.00E+00
C08	3.67E+00±1.84E+01－	5.57E-30±2.79E-29－	0.00E+00±0.00E+00
C09	3.83E+13±1.10E+13－**(4%)***	1.02E+13±5.24E+12－**(0%)***	4.30E+01±3.27E+01
C10	3.23E+13±1.17E+12－**(8%)***	1.11E+13±6.95E+12－**(4%)***	3.13E+01±8.22E-02
C11	-3.92E-04±2.64E-06≈	-1.40E+01±3.00E+00－**(0%)***	-3.92E-04±2.64E-06
C12	6.37E+01±2.81E+02－**(28%)***	-5.46E+03±7.19E+02－**(0%)***	-1.99E-01±1.42E-06
C13	-6.23E+01±1.50E+00－	-7.48E+01±8.66E+00－**(0%)***	-6.83E+01±1.95E-01
C14	3.56E+07±1.23E+08－	1.47E+01±4.84E+00－	9.80E-29±4.90E-28
C15	1.66E+14±4.79E+13－	2.74E+13±1.22E+13－	2.16E+01±8.03E-05
C16	1.18E+00±3.46E-02－**(40%)***	1.06E+00±2.57E-02－**(32%)***	0.00E+00±0.00E+00
C17	1.76E+03±4.89E+02－**(76%)***	6.63E+02±2.83E+02－**(44%)***	1.59E-01±3.82E-01
C18	2.87E+04±5.26E+03－	1.44E+04±6.15E+03－**(92%)***	4.87E-01±1.25E+00
－	16	16	
＋	0	0	
≈	2	2	

"－", "＋", and "≈" denote that the performance of the corresponding method is worse than, better than, and similar to that of FROFI, respectively. "*" denotes that feasible solutions cannot be consistently found by the corresponding method in all runs, and (#) denotes the feasible rate.

TABLE S4

COMPARISON OF FROFI WITH RESPECT TO εDE [47], APF-GA [69], (μ+λ)-CDE [31], DYHF [56], AND CMODE [57] IN TERMS OF THE SUCCESS RATE. IN ALL THE EXPERIMENTS, 25 INDEPENDENT RUNS WERE IMPLEMENTED ON 24 TEST FUNCTIONS FROM IEEE CEC2006 USING 5×10^5 FES.

Test Function from IEEE CEC2006	Success Rate					
	εDE	APF-GA	(μ+λ)-CDE	DyHF	CMODE	FROFI
g01	100%	100%	100%	100%	100%	100%
g02	100%	60%	96%	100%	100%	100%
g03	100%	100%	100%	100%	100%	100%
g04	100%	100%	100%	100%	100%	100%
g05	100%	55%	100%	100%	100%	100%
g06	100%	100%	100%	100%	100%	100%
g07	100%	100%	100%	100%	100%	100%
g08	100%	100%	100%	100%	100%	100%
g09	100%	100%	100%	100%	100%	100%
g10	100%	55%	100%	100%	100%	100%
g11	100%	100%	100%	100%	100%	100%
g12	100%	100%	100%	100%	100%	100%
g13	100%	70%	100%	100%	100%	100%
g14	100%	64%	100%	100%	100%	100%
g15	100%	100%	100%	100%	100%	100%
g16	100%	80%	100%	100%	100%	100%
g17	100%	36%	100%	100%	100%	100%
g18	100%	96%	100%	100%	100%	100%
g19	100%	100%	100%	100%	100%	100%
g20	0%	0%	100%	0%	100%	100%
g21	100%	0%	92%	100%	80%	100%
g22	0%	0%	0%	0%	0%	0%
g23	100%	0%	100%	100%	100%	100%
g24	100%	100%	100%	100%	100%	100%
Mean	91.67%	71.50%	95.33%	91.67%	95.00%	**95.83%**

TABLE S5

Comparison of FROFI with Respect toεDE [47], APF-GA [69], (μ+λ)-CDE [31], DyHF [56], and CMODE [57] in Terms of the Success Performance. In All the Experiments, 25 Independent runs were Implemented on 24 Test Functions from IEEE CEC2006 Using 5×10^5 FEs.

Test Function from IEEE CEC2006	Success Performance					
	εDE	APF-GA	(μ+λ)-CDE	DyHF	CMODE	FROFI
g01	5.9E+04	4.2E+05	8.9E+04	6.9E+04	1.2E+05	3.8E+04
g02	1.5E+05	6.8E+05	2.7E+05	1.1E+05	1.9E+05	8.5E+04
g03	8.9E+04	2.3E+05	1.1E+05	4.3E+04	7.5E+04	6.3E+04
g04	2.6E+04	2.6E+05	3.0E+04	4.0E+04	7.3E+04	2.5E+04
g05	9.7E+04	5.7E+05	1.6E+05	4.7E+04	2.9E+04	3.1E+04
g06	7.4E+03	2.0E+05	1.1E+04	3.8E+04	3.5E+04	1.5E+04
g07	7.4E+04	2.2E+05	1.4E+05	9.4E+04	1.6E+05	7.2E+04
g08	1.1E+03	5.7E+04	2.0E+03	1.2E+03	5.9E+03	2.4E+03
g09	2.3E+04	5.3E+04	4.0E+04	4.1E+04	7.1E+04	3.2E+04
g10	1.1E+05	2.9E+05	1.8E+05	1.4E+05	1.8E+05	1.0E+05
g11	1.6E+04	2.2E+05	7.9E+04	5.8E+03	6.0E+03	1.2E+04
g12	4.1E+03	1.1E+05	4.9E+03	3.0E+03	5.0E+03	3.5E+03
g13	3.5E+04	1.1E+05	1.4E+05	3.2E+04	3.1E+04	4.1E+04
g14	1.1E+05	2.4E+05	1.7E+05	6.5E+04	1.1E+05	6.6E+04
g15	8.4E+04	1.8E+04	1.3E+05	2.3E+04	1.3E+04	1.9E+04
g16	1.3E+04	4.9E+04	1.9E+04	3.0E+04	2.9E+04	1.8E+04
g17	9.9E+04	3.2E+05	1.8E+05	2.1E+05	1.4E+05	1.3E+05
g18	5.9E+04	2.2E+05	2.1E+05	8.9E+04	1.1E+05	9.6E+04
g19	3.5E+05	2.6E+04	2.6E+05	1.1E+05	2.5E+05	1.2E+05
g20	*NA*	*NA*	1.4E+05	*NA*	4.4E+05	4.7E+05
g21	1.4E+05	*NA*	2.1E+05	1.0E+05	1.3E+05	9.4E+04
g22	*NA*	*NA*	*NA*	*NA*	*NA*	*NA*
g23	2.0E+05	*NA*	2.6E+05	1.6E+05	2.4E+05	1.7E+05
g24	3.0E+03	1.9E+05	5.0E+03	1.4E+04	2.2E+04	5.7E+03

TABLE S6

Experimental Results of εDEag [51], SRS-εDEag [50], ECHT-DE [72], AIS-IRP [40], and FROFI over 25 Independent Runs on 18 Test Functions with 10D from IEEE CEC2010 Using 2×10^5 FEs. "Mean OFV" and "Std Dev" Indicate the Average and Standard Deviation of the Objective Function Values Obtained in 25 Runs, Respectively. t -Test at a 0.05 Significance Level was Performed between FROFI and Each of εDEag, SRS-εDEag, ECHT-DE, and AIS-IRP.

Test Function with 10D from IEEE CEC2010	εDEag	SRS-εDEag	ECHT-DE	AIS-IRP	FROFI
	Mean OFV±Std Dev	Mean OFV±Std Dev	Mean OFV±Std Dev	Mean OFV±Std Dev	Mean OFV±Std Dev
C01	-7.47E-01±1.32E-03≈	-7.47E-01±2.82E-03≈	-7.47E-01±1.40E-03≈	-7.47E-01±1.30E-03≈	-7.47E-01±1.35E-03
C02	-2.26E+00±2.39E-02＋	-2.27E+00±1.28E-02＋	-2.27E+00±6.70E-03＋	-2.27E+00±2.00E-03＋	-2.02E+00±1.41E-01
C03	0.00E+00±0.00E+00≈	0.00E+00±0.00E+00≈	0.00E+00±0.00E+00≈	3.75E-09±4.81E-04－	0.00E+00±0.00E+00
C04	-9.92E-06±1.55E-07－	-9.99E-06±1.60E-10－	-1.00E-05±0.00E+00≈	-9.97E-06±4.28E-03≈	-1.00E-05±0.00E+00
C05	-4.84E+02±3.89E-13≈	-4.84E+02±6.25E-13≈	-4.11E+02±7.63E+01－	-4.80E+02±6.30E+00－	-4.84E+02±8.09E-07
C06	-5.79E+02±3.63E-03－	-5.79E+02±9.07E-04≈	-5.62E+02±4.51E+01－	-5.80E+02±7.30E-08＋	-5.79E+02±5.04E-04
C07	0.00E+00±0.00E+00≈	0.00E+00±0.00E+00≈	1.33E-01±7.28E-01－	1.17E-08±2.70E+00－	0.00E+00±0.00E+00
C08	6.73E+00±5.56E+00≈	8.39E+00±4.49E+00≈	6.16E+00±6.45E+00≈	4.09E+00±1.46E+00＋	7.11E+00±4.79E+00
C09	0.00E+00±0.00E+00＋	2.93E+01±1.99E+01≈	1.47E-01±8.05E-01＋	2.70E+01±7.50E+01≈	2.50E+01±3.92E+01
C10	0.00E+00±0.00E+00＋	4.80E+01±3.29E+01≈	1.71E+00±7.66E+00＋	1.62E+03±5.00E+02－	4.17E+01±8.69E-06
C11	-1.52E-03±6.34E-11≈	-1.52E-03±6.02E-11≈	-4.40E-03±1.57E-02－*	-9.20E-04±8.23E-04－	-1.52E-03±5.63E-14
C12	-3.37E+02±1.78E+02－	-4.70E+02±1.42E+02＋	-1.72E+02±2.21E+02－*	-4.36E+02±6.02E+01≈	-3.84E+02±2.17E+02
C13	-6.84E+01±1.03E-06≈	-6.80E+01±1.33E+00－	-6.51E+01±2.38E+00－	-6.79E+01±3.11E-01－	-6.84E+01±2.52E-09
C14	0.00E+00±0.00E+00≈	0.00E+00±0.00E+00≈	7.02E+05±3.19E+06－	1.22E-04±2.90E-08－	0.00E+00±0.00E+00
C15	1.80E-01±8.81E-01＋	2.21E+01±1.05E+02－	2.34E+13±5.30E+13－	5.19E-09±1.10E-08＋	3.09E+00±1.37E+00
C16	3.70E-01±3.71E-01－	2.34E-02±2.64E-02≈	3.93E-02±4.28E-02－	9.96E-18±6.27E-15＋	1.19E-02±2.07E-02
C17	1.25E-01±1.94E-01－	4.15E-02±1.24E-01≈	1.12E-01±3.32E-01－	2.93E+00±2.29E+00－	7.83E-02±2.25E-01
C18	9.68E-19±1.81E-18－	5.79E-17±2.24E-16－	0.00E+00±0.00E+00＋	1.66E+00±1.27E+00－	5.23E-26±1.71E-25
－	6	4	10	9	
＋	4	2	4	5	
≈	8	12	4	4	

"－", "＋", and "≈" denote that the performance of the corresponding method is worse than, better than, and similar to that of FROFI, respectively. "*" denotes that feasible solutions cannot be consistently found by the corresponding method in all runs.

TABLE S7

EXPERIMENTAL RESULTS OF DYHF [56], CMODE [57], AND FROFI OVER 25 INDEPENDENT RUNS ON 18 TEST FUNCTIONS WITH 10D FROM IEEE CEC2010 USING 2×10^5 FES. "MEAN OFV" A*ND "STD DEV" INDICATE THE AVERAGE AND STANDARD DEVIATION OF THE OBJECTIVE FUNCTION VALUES OBTAINED IN 25 RUNS, RESPECTIVELY. WILCOXON'S RANK SUM TEST AT A 0.05 SIGNIFICANCE LEVEL WAS PERFORMED BETWEEN FROFI AND EACH OF DYHF AND CMODE.

Test Function with 10D from IEEE CEC2010	DyHF Mean OFV±Std Dev	CMODE Mean OFV±Std Dev	FROFI Mean OFV±Std Dev
C01	-7.47E-01±1.9E-03≈	-7.47E-01±2.35E-13≈	-7.47E-01±1.35E-03
C02	-9.45E-01±1.15E+00－**(84%)***	-1.48E+00±4.88E-01－**(96%)***	-2.02E+00±1.41E-01
C03	9.66E+04±4.83E+05－**(92%)***	2.84E+00±4.23E+00－	0.00E+00±0.00E+00
C04	-1.00E-05±3.34E-13－	-9.99E-04±2.90E-08－	-1.00E-05±0.00E+00
C05	-2.56E+02±1.53E+02－	-4.50E+02±1.61E+02－**(84%)***	-4.84E+02±8.09E-07
C06	-5.66E+02±1.65E+01－	-5.78E+02±1.60E-02－	-5.79E+02±5.04E-04
C07	0.00E+00±0.00E+00≈	6.69E-15±8.95E-15－	0.00E+00±0.00E+00
C08	5.13E+00±5.45E+00+	8.94E+00±3.98E+00≈	7.11E+00±4.79E+00
C09	2.10E+11±6.85E+11－**(68%)***	2.13E+06±1.04E+07－**(96%)***	2.50E+01±3.92E+01
C10	4.06E+11±1.40E+12－**(72%)***	2.13E+06±1.04E+07－**(96%)***	4.17E+01±8.69E-06
C11	-8.18E-01±2.20E+00－**(0%)***	-7.7E-02±2.85E-02－**(12%)***	-1.52E-03±5.63E-14
C12	-4.87E+02±4.38E+02－**(20%)***	-6.14E+02±2.74E+02－**(60%)***	-3.84E+02±2.17E+02
C13	-6.84E+01±8.10E-06－	-5.79E+01±4.09E+00－	-6.84E+01±2.52E-09
C14	2.10E+01±1.05E+02－	8.18E-09±1.64E-08－	0.00E+00±0.00E+00
C15	2.28E+12±8.94E+12－	1.20E+02±3.48E+02－	3.09E+00±1.37E+00
C16	1.55E-01±2.25E-01－	6.82E-05±1.49E-04+	1.19E-02±2.07E-02
C17	2.40E+01±7.00E+01－**(96%)***	4.37E-02±1.12E-01≈	7.83E-02±2.25E-01
C18	5.18E+02±8.84E+02－**(96%)***	5.75E+00±2.64E+02－	5.23E-26±1.71E-25
－	15	14	
＋	1	1	
≈	2	3	

"－", "＋", and "≈" denote that the performance of the corresponding method is worse than, better than, and similar to that of FROFI, respectively. "*" denotes that feasible solutions cannot be consistently found by the corresponding method in all runs, and (#) denotes the feasible rate.

TABLE S8

EXPERIMENTAL RESULTS OFεDEAG [51], SRS-εDEAG [50], ECHT-DE [72], AIS-IRP [40], AND FROFI OVER 25 INDEPENDENT RUNS ON 18 TEST FUNCTIONS WITH 30D FROM IEEE CEC2010 USING 6×10^5 FES. "MEAN OFV" AND "STD DEV" INDICATE THE AVERAGE AND STANDARD DEVIATION OF THE OBJECTIVE FUNCTION VALUES OBTAINED IN 25 RUNS, RESPECTIVELY. *t* -TEST AT A 0.05 SIGNIFICANCE LEVEL WAS PERFORMED BETWEEN FROFI AND EACH OF εDEAG, SRS-εDEAG, ECHT-DE, AND AIS-IRP.

Test Function with 30D from IEEE CEC2010	εDEag	SRS-εDEag	ECHT-DE	AIS-IRP	FROFI
	Mean OFV±Std Dev	Mean OFV±Std Dev	Mean OFV±Std Dev	Mean OFV±Std Dev	Mean OFV±Std Dev
C01	-8.21E-01±7.10E-04≈	-8.21E-01±6.10E-04≈	-8.00E-01±1.79E-02－	-8.20E-01±3.25E-04≈	-8.21E-01±2.36E-03
C02	-2.15E+00±1.20E-02＋	-2.19E+00±8.88E-03＋	-1.99E+00±2.10E-01－	-2.21E+00±2.84E-03＋	-2.00E+00±4.35E-02
C03	2.88E+01±8.05E-01－	2.87E+01±2.80E-07≈	9.89E+01±6.26E+01－	6.68E+01±4.26E+02－	2.87E+01±6.24E-08
C04	8.16E-03±3.07E-03－	5.70E-03±1.84E-03－	-1.03E-06±9.01E-03－	1.98E-03±1.61E-03－	-3.33E-06±4.13E-10
C05	-4.50E+02±2.90E+00－	-4.63E+02±3.37E+00－	-1.06E+02±1.67E+02－	-4.36E+02±2.51E+01－	-4.81E+02±2.84E+00
C06	-5.28E+02±4.75E-01－	-5.29E+02±2.54E-01≈	-1.38E+02±9.89E+01－	-4.54E+02±4.79E+01－	-5.29E+02±5.71E-01
C07	2.60E-15±1.23E-15－	2.70E-15±1.61E-15－	1.33E-01±7.28E-01－	1.07E+00±1.61E+00－	0.00E+00±0.00E+00
C08	7.83E-14±4.86E-14－	4.90E-14±3.09E-14－	3.36E+01±1.11E+02－	1.65E+00±6.41E-01－	0.00E+00±0.00E+00
C09	1.07E+01±2.82E+01＋	2.43E+00±1.20E+01＋	4.24E+01±1.38E+02≈	1.57E+00±1.96E+00＋	4.30E+01±3.27E+01
C10	3.33E+01±4.55E-01－	3.29E+01±4.74E-01－	5.34E+01±8.83E+01≈	1.78E+01±1.88E+01＋	3.13E+01±8.22E-02
C11	-2.86E-04±2.71E-05－	-2.99E-04±3.32E-05－	2.60E-03±6.00E-03－*	-1.58E-04±4.67E-05－	-3.92E-04±2.64E-06
C12	3.56E+02±2.89E+02－*	2.13E+02±2.71E+02－*	-2.51E+01±1.37E+02－*	4.29E-06±4.52E-04－	-1.99E-01±1.42E-06
C13	-6.54E+01±5.73E-01－	-6.59E+01±6.18E-01－	-6.46E+01±1.67E+00－	-6.62E+01±2.27E-01－	-6.83E+01±1.95E-01
C14	3.09E-13±5.61E-13－	1.04E-13±8.24E-14－	1.24E+05±6.77E+05－	8.68E-07±3.14E-07－	9.80E-29±4.90E-28
C15	2.16E+01±1.10E-04≈	2.16E+01±6.24E-05≈	1.94E+11±4.35E+11－	3.41E+01±3.82E+01－	2.16E+01±8.03E-05
C16	2.17E-21±1.06E-20－	0.00E+00±0.00E+00≈	0.00E+00±0.00E+00≈	8.21E-02±1.12E-01－	0.00E+00±0.00E+00
C17	6.33E+00±4.99E+00－	1.17E-01±7.77E-01≈	2.75E-01±3.78E-01－	3.61E+00±2.54E+00－	1.59E-01±3.82E-01
C18	8.75E+01±1.66E+02－	3.95E+01±6.23E+01－	0.00E+00±0.00E+00＋	4.02E+01±1.80E+01－	4.87E-01±1.25E+00
－	14	10	14	14	
＋	2	2	1	3	
≈	2	6	3	1	

"－", "＋", and "≈" denote that the performance of the corresponding method is worse than, better than, and similar to that of FROFI, respectively. "*" denotes that feasible solutions cannot be consistently found by the corresponding method in all runs.

TABLE S9

EXPERIMENTAL RESULTS OF DYHF [56], CMODE [57], AND FROFI OVER 25 INDEPENDENT RUNS ON 18 TEST FUNCTIONS WITH 30D FROM IEEE CEC2010 USING 6×10^5 FES. "MEAN OFV" AND "STD DEV" INDICATE THE AVERAGE AND STANDARD DEVIATION OF THE OBJECTIVE FUNCTION VALUES OBTAINED IN 25 RUNS, RESPECTIVELY. WILCOXON'S RANK SUM TEST AT A 0.05 SIGNIFICANCE LEVEL WAS PERFORMED BETWEEN FROFI AND EACH OF DYHF AND CMODE.

Test Function with 30D from IEEE CEC2010	DyHF Mean OFV±Std Dev	CMODE Mean OFV±Std Dev	FROFI Mean OFV±Std Dev
C01	-8.21E-01±1.80E-03≈	-8.21E-01±3.3E-03≈	-8.21E-01±2.36E-03
C02	5.74E-01±1.59E+00－**(88%)***	9.75E-01±6.25E+01－	-2.00E+00±4.35E-02
C03	3.03E+12±8.18E+12－**(0%)***	2.18E+01±1.25E+01≈	2.87E+01±6.24E-08
C04	8.25E+00±7.15E+00－**(0%)***	6.72E-04±4.24E-04－	-3.33E-06±4.13E-10
C05	2.95E+12±8.18E+12－**(60%)***	2.77E+02±2.03E+02－**(0%)***	-4.81E+02±2.84E+00
C06	-2.10E+01±2.91E+02－**(88%)***	-4.96E+02±2.15E+02－**(0%)***	-5.29E+02±5.71E-01
C07	1.59E-01±7.97E-01－	5.24E-05±5.89E-05－	0.00E+00±0.00E+00
C08	4.72E+00±2.36E+01－	3.68E-01±2.62E-01－	0.00E+00±0.00E+00
C09	1.50E+13±1.57E+13－**(60%)***	1.72E+13±1.07E+13－**(0%)***	4.30E+01±3.27E+01
C10	1.57E+13±1.38E+13－**(44%)***	1.60E+13±7.00E+12－**(12%)***	3.13E+01±8.22E-02
C11	-1.68E-01±7.04E-01－**(0%)***	9.5E-03±9.7E-03－**(48%)***	-3.92E-04±2.64E-06
C12	-1.59E+01±3.87E+02－**(0%)***	-3.46E+00±7.35E+02－**(84%)***	-1.99E-01±1.42E-06
C13	-6.61E+01±1.91E+00－	-3.89E+01±2.17E+00－	-6.83E+01±1.95E-01
C14	2.41E+12±8.94E+12－	9.31E+00±2.46E+00－	9.80E-29±4.90E-28
C15	5.49E+13±7.61E+13－	1.51E+13±8.26E+12－	2.16E+01±8.03E-05
C16	7.41E-01±1.85E-01－	6.30E-02±2.72E-02－	0.00E+00±0.00E+00
C17	6.04E+02±4.92E+02－**(76%)***	3.12E+02±2.75E+02－**(80%)***	1.59E-01±3.82E-01
C18	1.18E+04±1.31E+04－**(76%)***	7.36E+03±3.12E+03－	4.87E-01±1.25E+00
－	17	16	
＋	0	0	
≈	1	2	

"－", "＋", and "≈" denote that the performance of the corresponding method is worse than, better than, and similar to that of FROFI, respectively. "*" denotes that feasible solutions cannot be consistently found by the corresponding method in all runs, and (#) denotes the feasible rate.

TABLE S10

Experimental Results of FROFI_WoR and FROFI over 25 Independent Runs on 18 Test Functions with 10D from IEEE CEC2010 Using 2×10^5 FEs. "Mean OFV" and "Std Dev" Indicate the Average and Standard Deviation of the Objective Function Values Obtained in 25 Runs, Respectively. Wilcoxon's Rank Sum Test at a 0.05 Significance Level was Performed between FROFI and FROFI_WoR.

Test Function with 10D from IEEE CEC2010	FROFI_WoR	FROFI
	Mean OFV±Std Dev	Mean OFV±Std Dev
C01	-7.47E-01±1.4E-03≈	-7.47E-01±1.35E-03
C02	2.30E+00±1.49E+00－**(88%)***	-2.02E+00±1.41E-01
C03	0.00E+00±0.00E+00≈	0.00E+00±0.00E+00
C04	-1.00E-05±0.00E+00≈	-1.00E-05±0.00E+00
C05	3.31E+02±1.74E+02－**(0%)***	-4.84E+02±8.09E-07
C06	2.76E+02±2.04E+02－**(0%)***	-5.79E+02±5.04E-04
C07	1.60E-01±7.97E-01≈	0.00E+00±0.00E+00
C08	8.84E+00±4.17E+00－	7.11E+00±4.79E+00
C09	1.04E+13±6.68E+12－**(8%)***	2.50E+01±3.92E+01
C10	7.48E+12±5.07E+12－**(0%)***	4.17E+01±8.69E-06
C11	-1.52E-03±2.80E-18＋	-1.52E-03±5.63E-14
C12	-1.07E+01±5.00E+01－**(80%)***	-3.84E+02±2.17E+02
C13	-6.84E+01±2.97E-14＋	-6.84E+01±2.52E-09
C14	9.95E+10±9.01E+10－	0.00E+00±0.00E+00
C15	2.76E+13±2.01E+13－	3.09E+00±1.37E+00
C16	1.05E+00±2.51E-02－**(56%)***	1.19E-02±2.07E-02
C17	4.83E+02±2.23E+02－**(52%)***	7.83E-02±2.25E-01
C18	9.97E+03±4.65E+03－**(88%)***	5.23E-26±1.71E-25
－	12	
＋	2	
≈	4	

"－", "＋", and "≈" denote that the performance of the corresponding method is worse than, better than, and similar to that of FROFI, respectively. "*" denotes that feasible solutions cannot be consistently found by the corresponding method in all runs, and (#) denotes the feasible rate.

TABLE S11

EXPERIMENTAL RESULTS OF FROFI_WOR AND FROFI OVER 25 INDEPENDENT RUNS ON 18 TEST FUNCTIONS WITH 30D FROM IEEE CEC2010 USING 6×10^5 FES. "MEAN OFV" AND "STD DEV" INDICATE THE AVERAGE AND STANDARD DEVIATION OF THE OBJECTIVE FUNCTION VALUES OBTAINED IN 25 RUNS, RESPECTIVELY. WILCOXON'S RANK SUM TEST AT A 0.05 SIGNIFICANCE LEVEL WAS PERFORMED BETWEEN FROFI AND FROFI_WOR.

Test Function with 30D from IEEE CEC2010	FROFI_WOR Mean OFV±Std Dev	FROFI Mean OFV±Std Dev
C01	-8.20E-01±2.1E-03≈	-8.21E-01±2.36E-03
C02	3.22E+00±5.32E-01－	-2.00E+00±4.35E-02
C03	3.26E+11±1.21E+12－	2.87E+01±6.24E-08
C04	-3.31E-06±6.87E-08－	-3.33E-06±4.13E-10
C05	5.06E+02±8.05E+01－**(0%)***	-4.81E+02±2.84E+00
C06	5.40E+02±6.19E+01－**(0%)***	-5.29E+02±5.71E-01
C07	0.00E+00±0.00E+00≈	0.00E+00±0.00E+00
C08	1.60E-01±7.97E-01－	0.00E+00±0.00E+00
C09	3.40E+13±1.04E+13－**(0%)***	4.30E+01±3.27E+01
C10	4.32E+13±1.40E+13－**(4%)***	3.13E+01±8.22E-02
C11	-3.92E-04±4.78E-09＋	-3.92E-04±2.64E-06
C12	-3.23E+01±1.95E+02－**(48%)***	-1.99E-01±1.42E-06
C13	-6.25E+01±1.75E+00－	-6.83E+01±1.95E-01
C14	2.65E+08±1.01E+09－	9.80E-29±4.90E-28
C15	1.63E+14±5.43E+13－	2.16E+01±8.03E-05
C16	1.14E+00±4.01E-02－**(44%)***	0.00E+00±0.00E+00
C17	1.61E+03±5.05E+02－**(88%)***	1.59E-01±3.82E-01
C18	2.92E+04±6.16E+03－	4.87E-01±1.25E+00
－	15	
＋	1	
≈	2	

"－", "＋", and "≈" denote that the performance of the corresponding method is worse than, better than, and similar to that of FROFI, respectively. "*" denotes that feasible solutions cannot be consistently found by the corresponding method in all runs, and (#) denotes the feasible rate.

TABLE S12

EXPERIMENTAL RESULTS OF FROFI_13, FROFI_14, FROFI_15, FROFI_16, AND FROFI_17 OVER 25 INDEPENDENT RUNS ON 18 TEST FUNCTIONS WITH 30D FROM IEEE CEC2010 USING 6×105 FES. "MEAN OFV" AND "STD DEV" INDICATE THE AVERAGE AND STANDARD DEVIATION OF THE OBJECTIVE FUNCTION VALUES OBTAINED IN 25 RUNS, RESPECTIVELY.

Test Function with 30D from IEEE CEC2010	FROFI_13	FROFI_14	FROFI_15	FROFI_16	FROFI_17
	Mean OFV±Std Dev	Mean OFV±Std Dev	Mean OFV±Std Dev	Mean OFV±Std Dev	Mean OFV±Std Dev
C01	-8.20E-01±2.29E-03	-8.20E-01±2.25E-03	-8.21E-01±2.36E-03	-8.20E-01±2.39E-03	-8.20E-01±1.73E-03
C02	-1.99E+00±4.79E-02	-1.99E+00±4.34E-02	-2.00E+00±4.35E-02	-2.01E+00±3.80E-02	-2.00E+00±4.13E-02
C03	2.87E+01±2.55E-08	2.87E+01±3.99E-08	2.87E+01±6.24E-08	2.75E+01±5.73E+00	2.87E+01±5.74E-08
C04	-3.33E-06±8.53E-11	-3.33E-06±5.35E-10	-3.33E-06±4.13E-10	-3.33E-06±1.88E-10	-3.33E-06±8.2E-11
C05	-4.81E+02±1.70E+00	-4.81E+02±1.10E+00	-4.81E+02±2.84E+00	-4.81E+02±1.33E+00	-4.81E+02±1.58E+00
C06	-5.27E+02±1.20E+00	-5.28E+02±8.70E-01	-5.29E+02±5.71E-01	-5.28E+02±7.85E-01	-5.28E+02±7.60E-01
C07	0.00E+00±0.00E+00	1.59E-01±7.97E-01	0.00E+00±0.00E+00	0.00E+00±0.00E+00	0.00E+00±0.00E+00
C08	3.67E+00±1.84E+01	8.49E+00±2.94E+01	0.00E+00±0.00E+00	0.00E+00±0.00E+00	3.67E+00±1.84E+01
C09	5.68E+01±1.12E+02	5.70E+01±2.84E+01	4.30E+01±3.27E+01	3.67E+01±4.23E+01	4.14E+01±3.80E+01
C10	3.17E+01±1.47E+00	3.14E+01±9.02E-01	3.13E+01±8.22E-02	3.13E+01±2.64E-02	3.13E+01±1.85E-02
C11	-3.92E-04±1.65E-06	-3.92E-04±1.18E-08	-3.92E-04±2.64E-06	-3.92E-04±1.77E-07	-3.92E-04±2.44E-07
C12	1.71E-01±1.33E+00 **(84%)***	-1.17E+01±5.88E+01 **(88%)***	-1.99E-01±1.42E-06	-7.92E-01±3.35E+00 **(92%)***	-5.08E+00±2.52E+01 **(92%)***
C13	-6.82E+01±4.86E-01	-6.80E+01±6.59E-01	-6.83E+01±1.95E-01	-6.82E+01±4.01E-01	-6.83E+01±3.73E-01
C14	1.59E-01±7.97E-01	0.00E+00±0.00E+00	9.80E-29±4.90E-28	0.00E+00±0.00E+00	0.00E+00±0.00E+00
C15	2.16E+01±3.53E-05	2.16E+01±5.61E-05	2.16E+01±8.03E-05	2.16E+01±6.87E-05	2.16E+01±1.02E-04
C16	0.00E+00±0.00E+00	6.78E-04±3.39E-03	0.00E+00±0.00E+00	5.10E-03±4.42E-01	2.49E-03±1.24E-02
C17	1.47E-01±2.40E-01	2.99E-01±3.94E-01	1.59E-01±3.82E-01	2.25E-01±4.42E-01	2.19E-01±4.57E-01
C18	1.90E+00±6.08E+00	9.68E-01±2.33E+00	4.87E-01±1.25E+00	6.86E-01±1.73E+00	1.12E+00±3.55E+00

"*" denotes that feasible solutions cannot be consistently found by the corresponding method in all runs, and (#) denotes the feasible rate.