Multi-Surrogate-Assisted Ant Colony Optimization for Expensive Optimization Problems with Continuous and Categorical Variables

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***Abstract*—As an effective optimization tool for expensive optimization problems, surrogate-assisted evolutionary algorithms (SAEAs) have been widely studied in recent years. However, most of current SAEAs are designed for continuous/combinatorial expensive optimization problems, which are not suitable for mixed-variable expensive optimization problems. This paper focuses on one kind of mixed-variable expensive optimization problems: expensive optimization problems with continuous and categorical variables (EOPCCVs). A multi-surrogate-assisted ant colony optimization algorithm (MiSACO) is proposed to solve EOPCCVs. MiSACO contains two main strategies: multi-surrogate-assisted selection and surrogate-assisted local search. In the former, radial basis function (RBF) and least-squares boosting tree (LSBT) are employed as the surrogate models. Afterward, three selection operators (i.e., RBF-based selection, LSBT-based selection, and random selection) are devised to select three solutions from the offspring solutions generated by ACO, with the aim of coping with different types of EOPCCVs robustly and preventing the algorithm from being misled by inaccurate surrogate models. In the latter, sequence quadratic optimization coupled with RBF is utilized to refine the continuous variables of the best solution founded so far. By combining these two strategies, MiSACO can solve EOPCCVs with limited function evaluations. Three sets of test problems and two real-world cases are used to verify the effectiveness of MiSACO. The results demonstrate that MiSACO performs well on solving EOPCCVs.**

***Index Terms*—Surrogate-assisted evolutionary algorithms, mixed-variable expensive optimization problems, ant colony optimization, continuous variables, categorical variables**

I. Introduction

A. Expensive Optimization Problems (EOPs) with Continuous and Categorical Variables

EOPs refer to the optimization problems with time-consuming objective functions and/or constraints. EOPs can be classified into three categories: continuous EOPs which contain only continuous variables, combinatorial EOPs which contain only discrete variables[1], and mixed-variable EOPs [8], [9] which contain both continuous and discrete variables [10], [11].

Manuscript received December 1, 2020; revised March 2, 2021. This work was supported in part by the National Natural Science Foundation of China under Grant 61673397 and Grant 61976225, and in part by the Beijing Advanced Innovation Center for Intelligent Robots and Systems under Grant 2018IRS06. *(Corresponding author: Yong Wang.)*

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[1]Discrete variables can be integer variables [1], categorical variables [2], [3], binary variables [4], [5], and sequential variables [6], [7].

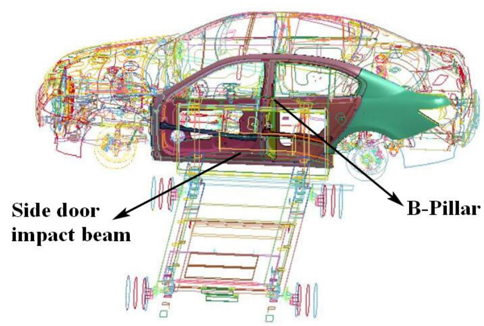

Fig. 1. The lightweight and crashworthiness design of the side body of an automobile [15].

Further, mixed-variable EOPs can be divided into different kinds according to the types of discrete variables, such as EOPs with continuous and integer variables [12], EOPs with continuous and binary variables [13], and EOPs with continuous and categorical variables (EOPCCVs) [14]. This paper mainly focuses on EOPCCVs.

In general, the mathematical model of an EOPCCV can be expressed as:

$$\begin{aligned} min:\ & f(\mathbf{x}^{cn},\mathbf{x}^{ca}) \\ s.t.\ & L_i^{cn} \le x_i^{cn} \le U_i^{cn} \\ & x_j^{ca} \in \mathbf{v}_j \end{aligned} \tag{1}$$

where $\mathbf{x}^{cn}=(x_1^{cn},x_2^{cn},\dots,x_{n_1}^{cn})$ and $\mathbf{x}^{ca}=(x_1^{ca},x_2^{ca},\dots,x_{n_2}^{ca})$ are the continuous and categorical vectors, respectively, n_1 is the number of continuous variables, n_2 is the number of categorical variables, $f(\mathbf{x}^{cn},\mathbf{x}^{ca})$ is the objective function, L_i^{cn} and U_i^{cn} are the lower and upper bounds of x_i^{cn}, respectively, $\mathbf{v}_j=\{v_j^1,v_j^2,\dots,v_j^{l_j}\}$ is the candidate categorical set for x_j^{ca}, and l_j is the size of $\mathbf{v}_j$.

Many real-world applications can be modeled as EOPCCVs [16], [17]. The lightweight and crashworthiness design of the side body of an automobile can be taken as an example [15], [18], as shown in Fig. 1. Usually, the side body of an automobile consists of many parts, such as B-Pillar and side door impact beam [19], [20]. Both the structure and material of each part have a great influence on the mass and carshworthiness. When designing the structure of each part, we need to consider its thickness, which is a continuous variable. In addition, we need to select a kind of material from the candidate material set for each part, which is a categorical variable. Moreover, the evaluation of carshworthiness is based on the finite element analysis (FEA), which is a time-consuming process. Therefore, it is an EOPCCV.

B. Surrogate-Assisted Evolutionary Algorithms (SAEAs)

As a kind of powerful optimization tool, EAs have been widely applied to solve science and engineering optimization problems [21]–[24]. However, since EAs usually need a large number of function evaluations (FEs) to obtain the optimal solution of an optimization problem, they are not suitable for EOPs. To overcome this barrier, SAEAs, which employ cheap surrogate models to replace a part of time-consuming exact FEs, have been developed [25]–[28]. In the past fifteen years, many SAEAs have been proposed to solve EOPs in different fields, such as mm-wave integrated circuit optimization [29], structure design of an automobile [30], trauma system design [13], neutral network architecture design [31], antenna design [32], and power system design [33].

Most of current SAEAs focus on continuous EOPs [34]–[37]), which utilize surrogate models for continuous functions, such as polynomial regression models (PRMs) [38], support vector regression [14], radial basis functions (RBFs) [39], artificial neural networks [40], and Gaussian processes (GPs) [41]. For example, Liu *et al*. [29] proposed a GP-assisted EA to deal with medium-scale EOPs. Tian *et al*. [42] adopted GP as the surrogate model, and proposed a multiobjective infill criterion to deal with high-dimensional EOPs. Wang *et al*. [30] proposed a global and local surrogate-assisted differential evolution for expensive constrained optimization problems. Sun *et al*. [43] proposed a surrogate-assisted cooperative swarm optimization algorithm to handle high-dimensional EOPs. Zhang *et al*. [44] combined MOEA/D [45] with GP to deal with expensive multiobjective optimization problems. Chugn *et al*. [46] proposed a surrogate-assisted reference vector guided EA to solve expensive many-objective optimization problems. Since different surrogate models for continuous functions have different strengths for different kinds of problem landscapes, many SAEAs with multiple or ensemble surrogate models for continuous functions are proposed [47]–[51]. For instance, Lim *et al*. [47] proposed a generalization of surrogate-assisted evolutionary frameworks. Le *et al*. [48] introduced an evolutionary framework with the evolvability learning of surrogates. Lu *et al*. [52] presented an evolutionary optimization framework with hierarchical surrogates. Li *et al*. [53] proposed an ensemble of surrogate assisted particle swarm optimization algorithm to solve medium-scale EOPs. Guo *et al*. [54] developed a multiobjective EA framework assisted by heterogeneous ensemble surrogate models.

Compared with continuous EOPs, few attempts have been made on combinatorial EOPs [55]. Current studies suggest that surrogate models with tree structures, such as random forest (RF) [56] and least-squares boosting tree (LSBT) [57], [58], are more suitable for dealing with combinatorial EOPs [55]. As a representative, Wang *et al*. [59] developed a RF-assisted EA for constrained multiobjective combinatorial optimization in trauma systems. Sun *et al*. [31] incorporated RF into a SAEA to design the architecture of convolutional neural networks. Moreover, some researchers incorporated domain knowledge into SAEAs to solve combinatorial EOPs, thus improving the search ability of the algorithms [60], [61].

Based on our investigation, only several papers work on EOPCCVs [14], [62], [63]. However, the methods proposed in these papers mainly extend surrogate models for continuous functions; thus, their capability of solving EOPCCVs is limited.

C. Motivation and Contributions

For SAEAs, the core problem is how to reasonably use surrogate models to guide the optimization process. As discussed in Section I-B, surrogate models for continuous functions are good at solving continuous EOPs and surrogate models with tree structures perform well on combinatorial EOPs. One may argue that EOPCCVs can be addressed by using surrogate models for continuous functions and surrogate models with tree structures to handle continuous and categorical variables, respectively. However, this way is unreasonable since continuous and categorical variables maybe interact with each other. Thus, they cannot be optimized separately.

Intuitively, the numbers of continuous and categorical variables have a significant impact on the performance of surrogate models. With respect to an EOPCCV, we consider the following three cases: most of its variables are continuous variables; most of its variables are categorical variables; and the number of continuous variables is similar to that of categorical variables. Obviously, surrogate models for continuous functions and surrogate models with tree structures are good choices for the first and second cases, respectively. However, for the third case, both of these two kinds of surrogate models are necessary. Note that even for the first and second cases, we cannot only use one of these two kinds of surrogate models due to the fact that EOPCCVs contain continuous and categorical variables at the same time. Therefore, in this paper, we employ these two kinds of surrogate models simultaneously.

The next issue which arises naturally is how to choose these two kinds of surrogate models for EOPCCVs. In this paper, RBF and LSBT are selected as the surrogate model for continuous functions and the surrogate model with a tree structure, respectively. The reasons for our selection are twofold: 1) RBF is a widely used surrogate model for continuous functions. It is simple and easy to train. Moreover, as a kernel-based model, RBF has the potential to deal with categorical variables through redefining the distance between two categorical vectors; and 2) As a kind of ensemble surrogate models, LSBT shows excellent generalization ability. In addition, LSBT has the potential to deal with continuous variables by discretizing the decision space. Therefore, they are expected to complement one another for solving EOPCCVs.

Subsequently, a multi-surrogate-assisted ant colony optimization (ACO) algorithm, called MiSACO, is proposed in this paper to solve EOPCCVs. To the best of our knowledge, MiSACO is the first attempt to incorporate both a surrogate model for continuous functions and a surrogate model with a tree structure into an EA to solve EOPCCVs. MiSACO introduces two important strategies: 1) multi-surrogate-assisted selection and 2) surrogate-assisted local search.

The main contributions of this paper can be summarized as follows:

- The aim of the multi-surrogate-assisted selection is to select promising solutions from the offspring solutions generated by ACO$_{MV}$ [10]. In this strategy, we select the best solution from the offspring solutions based on the predicted values provided by RBF (called RBF-based selection) and the best solution from the offspring solutions based on the predicted values provided by LSBT (called LSBT-based selection). In addition, to avoid the population being misled by inaccurate surrogate models, we also randomly select a solution from the offspring solutions (called random selection). As a result, three promising solutions are selected.
- The surrogate-assisted local search is designed to accelerate the convergence. In this strategy, if the number of evaluated solutions, which have the same categorical variables as the current best solution, is bigger than a threshold, these evaluated solutions are used to construct a RBF for only continuous variables. Based on the constructed RBF, the continuous variables of the current best solution are further optimized by using sequence quadratic programming (SQP), thus improving the quality of the current best solution quickly.
- Three sets of test problems are used to study the performance of MiSACO. The results suggest that MiSACO has the capability to cope with different types of EOPCCVs. We also apply MiSACO to two practical engineering design problems, i.e., the topographical design of stiffened plates against blast loading, and the lightweight and crashworthiness design for the side body of an automobile. The results show that MiSACO can effectively solve them.

The rest of this paper is organized as follows. Section II introduces the related techniques including the adopted surrogate models and search engine. Section III analyzes the characteristics of RBF and LSBT. The proposed algorithm, MiSACO, is elaborated in Section IV. The experimental studies are executed in Section V. In Section VI, MiSACO is applied to two engineering design problems in the real world. Finally, Section VII concludes this paper.

II. Related Techniques

A. RBF

As a commonly used surrogate model, RBF has been widely applied to approximate continuous functions in various science and engineering fields. Based on database $\{(\mathbf{x}_i, y_i)|i = 1,\ldots,N\}$, RBF approximates a continuous function as follows:

$$\hat{f}_{RBF}(\mathbf{x}) = \sum_{i=1}^{N} w_i \phi(dis(\mathbf{x},\mathbf{x}_i)) \tag{2}$$

where $dis(\mathbf{x},\mathbf{x}_i) = ||\mathbf{x}-\mathbf{x}_i||$ represents the Euclidean distance between $\mathbf{x}$ and $\mathbf{x}_i$, and w_i and $\phi(\cdot)$ are the weight coefficient and the basis function, respectively. In general, the Gaussian function [64] is employed as the basis function. When the least-squares loss is taken as the loss function, the weight vector $\mathbf{w} = (w_1, ..., w_N)$ can be calculated as follows:

Algorithm 1 ACO_{MV}

1: Initialize $\mathbb{SA}$;
2: **while** the termination criterion is not satisfied **do**
3: **for** $i = 1 : M$ **do**
4: Construct the ith offspring solutions based on the probabilities provided by (9) and (12);
5: Evaluate the ith offspring solutions;
6: **end for**
7: Update $\mathbb{SA}$ according to the generated M offspring solutions and elitist selection;
8: **end while**
9: Output the optimal solution

$$\mathbf{w} = (\Phi^T\Phi)^{-1}\Phi^T\mathbf{y} \tag{3}$$

where $\mathbf{y} = (y_1,\ldots,y_N)$ is the output vector and Φ is the matrix computed as follows:

$$\Phi = \begin{bmatrix} \phi(dis(\mathbf{x}_1,\mathbf{x}_1)) & \cdots & \phi(dis(\mathbf{x}_1,\mathbf{x}_N)) \\ \vdots & \ddots & \vdots \\ \phi(dis(\mathbf{x}_N,\mathbf{x}_1)) & \cdots & \phi(dis(\mathbf{x}_N,\mathbf{x}_N)) \end{bmatrix} \tag{4}$$

Note that, when approximating functions with both continuous and categorical variables, the distance between two solutions needs to be redefined. Inspired by Hamming distance, in this paper, the distance between the solution to be predicted (i.e., $(\mathbf{x}^{cn},\mathbf{x}^{ca})$) and the ith solution in the database (i.e., $(\mathbf{x}_i^{cn},\mathbf{x}_i^{ca})$) is calculated by

$$dis((\mathbf{x}^{cn},\mathbf{x}^{ca}),(\mathbf{x}_i^{cn},\mathbf{x}_i^{ca})) = \sqrt{||\mathbf{x}^{cn}-\mathbf{x}_i^{cn}||^2 + ||\mathbf{x}^{ca}\oplus\mathbf{x}_i^{ca}||^2} \tag{5}$$

where $||\cdot||$ represents the vector norm, $(\mathbf{x}^{cn}-\mathbf{x}_i^{cn})$ represents the difference of two continuous vectors (i.e., $\mathbf{x}^{cn}$ and $\mathbf{x}_i^{cn}$), and $(\mathbf{x}^{ca}\oplus\mathbf{x}_i^{ca})$ represents the vector after the xor operation of two categorical vectors (i.e., $\mathbf{x}^{ca}$ and $\mathbf{x}_i^{ca}$).

B. LSBT

LSBT is a kind of ensemble learning method based on binary regression trees [57]. The binary regression tree approximates a function by dividing the decision space into several subregions and each of them provides the same predicted value. LSBT can be expressed as the sum of several binary regression trees:

$$\hat{f}_{LSBT}(\mathbf{x}) = \sum_{m=1}^{M} T(\mathbf{x},\Theta_m) \tag{6}$$

where M is the total number of the binary regression trees, $T(\mathbf{x},\Theta_m)$ is the mth binary regression tree, and Θ_m represents the parameter vector of $T(\mathbf{x},\Theta_m)$ which can be determined iteratively.

Based on database $\{(\mathbf{x}_i,y_i)|i=1,\ldots,N\}$ and the residual of $\mathbf{x}_i$ in the mth iteration (denoted as $r_{m,i}$), Θ_m can be obtained by minimizing the following formulation:

$$\sum_{i=1}^{N}(r_{m,i}-T(\mathbf{x}_i,\Theta_m))^2 \tag{7}$$

where $r_{m,i} = y_i - \sum_{v=1}^{m-1} T(\mathbf{x}_i, \Theta_v)$.

Note that, to cope with continuous variables, when training LSBT, several discrete points are provided for each dimension to divide the decision space into several discrete subregions. When predicting the function value of a solution, these discrete points are used to determine which subspace the solution to be predicted is in.

C. ACO$_{MV}$

The process of ACO$_{MV}$ [10] is described in **Algorithm 1**. First, we initialize a solution archive (denoted as $\mathbb{SA}$), the purpose of which is to store the continuous and categorical variables of the best K evaluated solutions. The sth ($s \in \{1,\ldots,K\}$) solution in $\mathbb{SA}$ is denoted as: $\mathbf{SA}_s = [\mathbf{SA}_s^{cn}, \mathbf{SA}_s^{ca}] = [(SA_{s,1}^{cn}, SA_{s,2}^{cn}, \ldots, SA_{s,n_1}^{cn}), (SA_{s,1}^{ca}, SA_{s,2}^{ca}, \ldots, SA_{s,n_2}^{ca})]$. A weight (denoted as α_s) is then associated with $\mathbf{SA}_s$, which is calculated as:

$$\alpha_s = \frac{1}{qK\sqrt{2\pi}} e^{\frac{-(rank_s - 1)^2}{2q^2K^2}}, \ s \in \{1,\ldots,K\} \tag{8}$$

where $rank_s$ represents the rank of $\mathbf{SA}_s$, and q is a parameter called the influence of the best-quality solutions. Note that, by utilizing (8), the best solution receives the highest weight, while the weights of the other solutions decrease exponentially with their ranks. Next, ACO$_{MV}$ generates M offspring solutions at each iteration and the elitist selection is used to update $\mathbb{SA}$. The offspring solutions are generated according to α_s. A solution with a big α_s value means a higher probability of sampling around this solution. Since solutions with big α_s values have good objective function values, generating offspring solutions in such a way tends to make the algorithm converge to the promising region. Finally, when the termination criterion is satisfied, the obtained optimal solution is output.

The continuous and categorical variables of an offspring solution are generated in the following ways:

- When generating the continuous variables of an offspring solution, the continuous vector of the sth solution in $\mathbb{SA}$ is selected based on the following probability:

$$p_s = \frac{\alpha_s}{\sum_{r=1}^{K} \alpha_r}, \ s \in \{1,\ldots,K\} \tag{9}$$

  We denote the selected continuous vector as $\mathbf{S}^{cn} = (S_1^{cn}, S_2^{cn}, \ldots, S_{n_1}^{cn})$. Then, the ith continuous variable of an offspring solution is generated according to the following Gaussian probability density function:

$$g(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \tag{10}$$

  where $\mu = S_i^{cn}$. In (10), σ is calculated as:

$$\sigma = \xi \sum_{j=1}^{K} \frac{|SA_{j,i}^{cn} - S_i^{cn}|}{K-1} \tag{11}$$

  where ξ is a parameter called the width of search.
- For the jth categorical variable of an offspring solution, it is chosen from $\mathbf{v}_j = \{v_j^1, ..., v_j^{l_j}\}$ with the following probability:

$$p_{jt} = \frac{\beta_t}{\sum_{h=1}^{l_j} \beta_h}, \ t \in \{1,\ldots,l_j\} \tag{12}$$

  where β_t is the weight associated with the tth value. It is obvious that there are l_j values for the jth categorical variable. Suppose that η_j is the number of values that do not appear in $\mathbb{SA}$, and u_{jt} is the repeated number of the tth value that appears in $\mathbb{SA}$. If $u_{jt} > 1$, suppose that the indexes of the weights corresponding to the tth value in $\mathbb{SA}$ are: $id_1, \ldots, id_{u_{jt}}$. Let $\alpha_{jt} = \max\{\alpha_{id_1}, \ldots, \alpha_{id_{u_{jt}}}\}$. Then, β_t is calculated as:

$$\beta_t = \begin{cases} \frac{\alpha_{jt}}{u_{jt}} + \frac{q}{\eta_j}, & if(\eta_j > 0, \ u_{jt} > 0) \\ \frac{q}{\eta_j}, & if(\eta_j > 0, \ u_{jt} = 0) \\ \frac{\alpha_{jt}}{u_{jt}}, & if(\eta_j = 0, \ u_{jt} > 0) \end{cases} \tag{13}$$

III. Characteristics of RBF and LSBT

In this section, the characteristics of RBF and LSBT on approximating EOPCCVs are analyzed. First, three assumptions are given. Afterward, we analyze the predicted error of RBF and LSBT based on these three assumptions. Finally, some considerations behind the analysis are provided.

A. Assumptions

Firstly, we give an assumption to limit the change range of the objective function value according to the distance between two solutions. This assumption is inspired by the bi-Lipschitz continuity, i.e., a kind of smoothness condition that has been widely used in the theoretical analysis of Bayesian optimization [65]. The assumption is provided as follows.

Assumption 1: Based on the distance defined in Section II-A, $f(\mathbf{x}^{cn}, \mathbf{x}^{ca})$ and $\hat{f}_{RBF}(\mathbf{x}^{cn}, \mathbf{x}^{ca})$ satisfy the following conditions:

$$\begin{aligned} &|f(\mathbf{x}_1^{cn}, \mathbf{x}_1^{ca}) - f(\mathbf{x}_2^{cn}, \mathbf{x}_2^{ca})| \geq \frac{1}{L_f} \cdot dis((\mathbf{x}_1^{cn}, \mathbf{x}_1^{ca}), (\mathbf{x}_2^{cn}, \mathbf{x}_2^{ca})) \\ &|f(\mathbf{x}_1^{cn}, \mathbf{x}_1^{ca}) - f(\mathbf{x}_2^{cn}, \mathbf{x}_2^{ca})| \leq L_f \cdot dis((\mathbf{x}_1^{cn}, \mathbf{x}_1^{ca}), (\mathbf{x}_2^{cn}, \mathbf{x}_2^{ca})) \\ &|\hat{f}_{RBF}(\mathbf{x}_1^{cn}, \mathbf{x}_1^{ca}) - \hat{f}_{RBF}(\mathbf{x}_2^{cn}, \mathbf{x}_2^{ca})| \geq \frac{1}{L_r} \cdot dis((\mathbf{x}_1^{cn}, \mathbf{x}_1^{ca}), (\mathbf{x}_2^{cn}, \mathbf{x}_2^{ca})) \\ &|\hat{f}_{RBF}(\mathbf{x}_1^{cn}, \mathbf{x}_1{}^{ca}) - \hat{f}_{RBF}(\mathbf{x}_2^{cn}, \mathbf{x}_2^{ca})| \leq L_r \cdot dis((\mathbf{x}_1^{cn}, \mathbf{x}_1^{ca}), (\mathbf{x}_2^{cn}, \mathbf{x}_2^{ca})) \end{aligned} \tag{14}$$

where $(\mathbf{x}_1^{cn}, \mathbf{x}_1^{ca})$ and $(\mathbf{x}_2^{cn}, \mathbf{x}_2^{ca})$ are two different solutions, and L_f and L_r are two parameters [65].

Secondly, we consider that, for at least one solution, RBF and LSBT can accurately predict its objective function value. Thus, the following assumption is provided.

Assumption 2: For both RBF and LSBT, there exists at least one reference solution $(\mathbf{x}_*^{cn}, \mathbf{x}_*^{ca})$ that can make $f(\mathbf{x}_*^{cn}, \mathbf{x}_*^{ca}) = \hat{f}_{RBF}(\mathbf{x}_*^{cn}, \mathbf{x}_*^{ca}) = \hat{f}_{LSBT}(\mathbf{x}_*^{cn}, \mathbf{x}_*^{ca})$.

Finally, to make it easier to analyze the predicted error of LSBT, the following assumption is given.

Assumption 3: When predicting the objective function value, we assume that the solution to be predicted (denoted as $(\mathbf{x}^{cn}, \mathbf{x}^{ca})$) is close to $(\mathbf{x}_*^{cn}, \mathbf{x}_*^{ca})$; thus, $(\mathbf{x}^{cn}, \mathbf{x}^{ca})$ and $(\mathbf{x}_*^{cn}, \mathbf{x}_*^{ca})$ are located in the same subregion provided by LSBT.

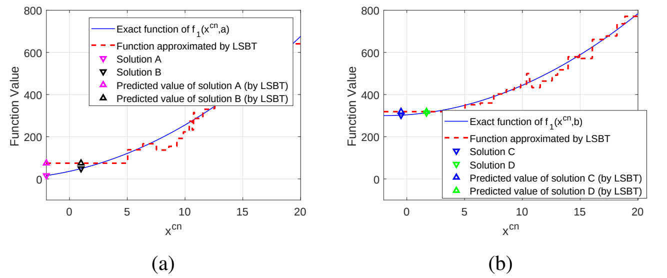

Fig. 2. Approximate $f_1(x^{cn}, x^{ca})$ by LSBT: (a) The landscapes of the exact function of $f_1(x^{cn}, x^{ca})$ and the function approximated by LSBT when $x^{ca} = a$. (b) The landscapes of the exact function of $f_1(x^{cn}, x^{ca})$ and the function approximated by LSBT when $x^{ca} = b$.

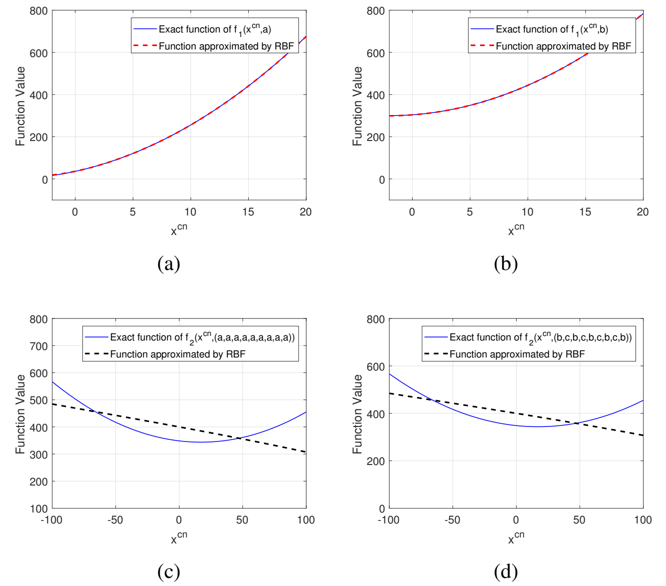

Fig. 3. Approximate $f_1(x^{cn}, x^{ca})$ and $f_2(x^{cn}, \mathbf{x}^{ca})$ by RBF: (a) The landscapes of the exact function of $f_1(x^{cn}, x^{ca})$ and the function approximated by RBF when $x^{ca} = a$. (b) The landscapes of the exact function of $f_1(x^{cn}, x^{ca})$ and the function approximated by RBF $x^{ca} = b$. (c) The landscapes of the exact function of $f_2(x^{cn}, \mathbf{x}^{ca})$ and the function approximated by RBF when $\mathbf{x}^{ca} = (a,a,a,a,a,a,a,a,a)$. (d) The landscapes of the exact function of $f_2(x^{cn}, \mathbf{x}^{ca})$ and the function approximated by RBF when $\mathbf{x}^{ca} = (b,c,b,c,b,c,b,c,b)$.

B. Analysis

For RBF and LSBT, we have the following two propositions.

Proposition 1: The upper and lower bounds of RBF's predicted error are described as:

$$|f(\mathbf{x}^{cn}, \mathbf{x}^{ca}) - \hat{f}_{RBF}(\mathbf{x}^{cn}, \mathbf{x}^{ca})| \leq (L_f + L_r) \cdot dis((\mathbf{x}^{cn}, \mathbf{x}^{ca}), (\mathbf{x}_*^{cn}, \mathbf{x}_*^{ca})) \tag{15}$$

Proposition 2: The upper and lower bounds of LSBT's predicted error are described as:

$$|f(\mathbf{x}^{cn}, \mathbf{x}^{ca}) - \hat{f}_{LSBT}(\mathbf{x}^{cn}, \mathbf{x}^{ca})| \leq L_f \cdot dis((\mathbf{x}^{cn}, \mathbf{x}^{ca}), (\mathbf{x}_*^{cn}, \mathbf{x}_*^{ca})) \tag{16}$$

and

$$|f(\mathbf{x}^{cn}, \mathbf{x}^{ca}) - \hat{f}_{LSBT}(\mathbf{x}^{cn}, \mathbf{x}^{ca})| \geq \frac{1}{L_f} \cdot dis((\mathbf{x}^{cn}, \mathbf{x}^{ca}), (\mathbf{x}_*^{cn}, \mathbf{x}_*^{ca})) \tag{17}$$

The proof of Proposition 1 and Proposition 2 is given in Section S-I of the supplementary file. Next, based on these two propositions, we discuss the upper and lower bounds of the predicted error.

1) Discussions about the upper bound: From (15) and (16), it can be observed that, when $dis((\mathbf{x}^{cn}, \mathbf{x}^{ca}), (\mathbf{x}_*^{cn}, \mathbf{x}_*^{ca})) \neq 0$, the upper bound of LSBT's predicted error is always smaller than that of RBF's.

2) Discussions about the lower bound: It should be noted that the best predicted error provided by RBF can be equal to zero, which means that RBF has the chance to provide an accurate predicted value. In contrast, according to (17), the lower bound of LSBT's predicted error cannot be zero when $dis((\mathbf{x}^{cn}, \mathbf{x}^{ca}), (\mathbf{x}_*^{cn}, \mathbf{x}_*^{ca})) \neq 0$, which means that it is hard for LSBT to accurately predict the objective function values of all solutions except the reference solution.

In summary, RBF and LSBT have different advantages and disadvantages. When copping with EOPCCVs, RBF has the opportunity to accurately predict the objective function value of each solution. However, the upper bound of its predicted error is larger than that of LSBT. Thus, compared with LSBT, the predicted error of RBF may fluctuate greatly. In contrast, under the condition that $dis((\mathbf{x}^{cn}, \mathbf{x}^{ca}), (\mathbf{x}_*^{cn}, \mathbf{x}_*^{ca})) \neq 0$, it is hard for LSBT to accurately predict the objective function, but LSBT has a stable prediction capability.

C. Considerations based on the Above Analysis

According to the above analysis, we have the following considerations about how to effectively use surrogate models when solving EOPCCVs:

- Due to the fact that LSBT has a stable prediction capability, when using it to select a solution from the offspring solutions, the one with good quality is very likely to be chosen. However, since it is hard for LSBT to accurately approximate the objective function of an EOPCCV, the most promising solution may be missed. Therefore, only using LSBT to guide the algorithm may not be effective. An example in Fig. 2 is used to illustrate this issue. In Fig. 2, LSBT is used to approximate $f_1(x^{cn}, x^{ca})$ with $n_1 = 1$, $n_2 = 1$, $x^{cn} \in [-2, 20]$, and $x^{ca} \in \{a, b\}$. The aim is to select the best solution from **A**, **B**, **C**, and **D**. Note that, **A** has the best original objective function value, and **A** and **B** are better than **C** and **D**. According to the predict values, the solution with good quality (i.e., **A** or **B**) will be selected. However, since **A** and **B** have the same predict value, the most promising solution (i.e., **A**) may not be selected.
- Although RBF has the opportunity to accurately approximate the objective function of each solution, its prediction error may fluctuate greatly. Therefore, only using RBF to guide the search may mislead the algorithm to converge to a wrong optimal solution. An example in Fig. 3 is used to illustrate this issue. In Fig. 3, RBF is used to approximate the following two functions: $f_1(x^{cn}, x^{ca})$ with $n_1 = 1$, $n_2 = 1$, $x^{cn} \in [-2, 20]$, and $x^{ca} \in \{a, b\}$, and $f_2(x^{cn}, \mathbf{x}^{ca})$ with $n_1 = 1$, $n_2 = 9$, $x^{cn} \in [-100, 100]$, and $x_j^{ca} \in \{a, b, c, d, e\} (j = \{1, \ldots, n_2\})$. The exact functions of $f_1(x^{cn}, x^{ca})$ and $f_2(x^{cn}, \mathbf{x}^{ca})$, and the functions approx-

Algorithm 2 *MiSACO*

1: [$\mathbb{SA}$, $\mathbb{DB}$, *FEs*] $\leftarrow$ *Initialization*;
2: **while** $FEs < MaxFEs$ **do**
3: $\quad\mathbb{OP} \leftarrow ACO_{MV}(\mathbb{SA})$;
4: $\quad\mathbb{X}_{sel} \leftarrow MSA\_Selection(\mathbb{OP})$;
5: $\quad\mathbb{X}_{ls} \leftarrow SA\_LocalSearch(\mathbb{DB})$;
6: $\quad\mathbb{X} = \mathbb{X}_{sel} \cup \mathbb{X}_{ls}$;
7: $\quad[\mathbb{Y}, FEs] \leftarrow Evaluation(\mathbb{X}, FEs)$;
8: $\quad\mathbb{DB} = \mathbb{DB} \cup \{\mathbb{X}, \mathbb{Y}\}$;
9: $\quad\mathbb{SA} \leftarrow Update(\mathbb{SA}, \mathbb{X}, \mathbb{Y})$;
10: **end while**
11: Output the best solution $\mathbf{x}_{best}$.

imated by RBF are exhibited in Fig. 3. For convenience, when approximating $f_2(x^{cn}, \mathbf{x}^{ca})$, we only exhibit the functions when $\mathbf{x}^{ca} = (a,a,a,a,a,a,a,a,a)$ and $\mathbf{x}^{ca} = (b,c,b,c,b,c,b,c,b)$. It can be observed that RBF can exactly approximate $f_1(x^{cn}, x^{ca})$. However, with respect to $f_2(x^{cn}, \mathbf{x}^{ca})$, the prediction error of RBF is large, thus may misleading the optimization process.

Based on the above considerations, we employ both LSBT and RBF to assist EAs to handle EOPCCVs. By doing this, on one hand, LSBT ensures that a solution with good quality can be selected from the offspring solutions generated by EAs; on the other hand, the use of RBF makes EAs have a chance to select the most promising solution, thus improving the efficiency of evolution.

IV. Proposed Method

A. General Framework

The framework of MiSACO is given in **Algorithm 2**. The symbols in **Algorithm 2** are explained as follows:

- $\mathbb{DB}$: the database containing the information of all the evaluated solutions, i.e., the continuous variables, the categorical variables, and the objective function values of all the evaluated solutions.
- $\mathbb{SA}$: the solution archive used in ACO_{MV}.
- *FEs*: the number of FEs.
- $\mathbb{OP}$: the set containing the offspring solutions generated by ACO_{MV}.
- $\mathbb{X}_{sel}$: the set containing the solutions selected by the multi-surrogate-assisted selection.
- $\mathbb{X}_{ls}$: the set containing the solution founded by the surrogate-assisted local search.
- $\mathbb{X}$: the set containing the solutions in $\mathbb{X}_{sel}$ and $\mathbb{X}_{ls}$.
- $\mathbb{Y}$: the set containing the original expensive objective function values of the solutions in $\mathbb{X}$.

The process of **Algorithm 2** can be divided into the following five steps:

- **Initialization** (Line 1): this step produces K initial solutions via Latin hypercube design and puts them into $\mathbb{SA}$. Subsequently, it evaluates them by the original expensive objective function and initializes $\mathbb{DB}$ and *FEs*.
- **Generating the offspring** (Line 3): In this step, M offspring solutions are generated based on ACO_{MV} and reserved into $\mathbb{OP}$.

Algorithm 3 *MSA_Selection*($\mathbb{OP}$)

Construct a RBF surrogate model (denoted as $\hat{f}_{RBF}$) and a LSBT surrogate model (denoted as $\hat{f}_{LSBT}$) by utilizing all the solutions in $\mathbb{DB}$ to approximate the objective function of an EOPCCV;
Evaluate the solutions in $\mathbb{OP}$ with $\hat{f}_{RBF}$ and select the best one, denoted as $\mathbf{x}_1$;
$\mathbb{OP} = \mathbb{OP} \setminus \mathbf{x}_1$;
Evaluate the solutions in $\mathbb{OP}$ with $\hat{f}_{LSBT}$ and select the best one, denoted as $\mathbf{x}_2$;
$\mathbb{OP} = \mathbb{OP} \setminus \mathbf{x}_2$;
Select a solution from $\mathbb{OP}$ randomly, denoted as $\mathbf{x}_3$;
$\mathbb{X}_{sel} = \{\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3\}$;
Output $\mathbb{X}_{sel}$.

- **Multi-Surrogate-Assisted Selection** (Line 4): This step selects three solutions from $\mathbb{OP}$ according to the multi-surrogate-assisted selection, and puts them into $\mathbb{X}_{sel}$.
- **Surrogate-Assisted Local Search** (Line 5): In this step, the surrogate-assisted local search is used to enhance the quality of the best solution in $\mathbb{DB}$ by further optimizing its continuous variables, and the obtained solution is reserved into $\mathbb{X}_{ls}$.
- **Updating $\mathbb{DB}$ and $\mathbb{SA}$** (Lines 6-9): In this step, the solutions in $\mathbb{X}_{sel}$ and $\mathbb{X}_{ls}$ are evaluated by the original expensive objective function. The information of them is kept in $\mathbb{DB}$. Then, based on these solutions, $\mathbb{SA}$ is updated according to the elitist selection.

The unique characteristic of MiSACO lies in its multi-surrogate-assisted selection and surrogate-assisted local search. Next, we explain these two strategies respectively.

B. Multi-Surrogate-Assisted Selection

The process of the multi-surrogate-assisted selection is described in **Algorithm 3**, in which three selection operators (i.e., the RBF-based selection, the LSBT-based selection, and the random selection) are adopted to select three promising solutions from $\mathbb{OP}$. Firstly, we construct a RBF surrogate model (denoted as $\hat{f}_{RBF}$) and a LSBT surrogate model (denoted as $\hat{f}_{LSBT}$) based on $\mathbb{DB}$, and evaluate all the solutions in $\mathbb{OP}$ by using $\hat{f}_{RBF}$ and $\hat{f}_{LSBT}$, respectively. Then, the solution with the best $\hat{f}_{RBF}$ value is selected from $\mathbb{OP}$. We record this solution as $\mathbf{x}_1$, and remove it from $\mathbb{OP}$. Subsequently, the solution with the best $\hat{f}_{LSBT}$ value, denoted as $\mathbf{x}_2$, is selected and removed from $\mathbb{OP}$. Next, a solution, denoted as $\mathbf{x}_3$, is randomly selected from $\mathbb{OP}$. Finally, these three solutions (i.e., $\mathbf{x}_1$, $\mathbf{x}_2$, and $\mathbf{x}_3$) are reserved into $\mathbb{X}_{sel}$.

As mentioned in Section III, when RBF or LSBT is used to approximate the objective function of an EOPCCV, the accuracy cannot be guaranteed. If the accuracy is poor, some solutions with good original objective function values may have bad predicted values. Under this condition, they may be missed. To alleviate this issue, we randomly select a solution from $\mathbb{OP}$ without depending on any surrogate model. An example in Fig. 4 is used to illustrate this issue: $f_3(x^{cn}, x^{ca})$ with $n_1 = 1$, $n_2 = 1$, $x^{cn} \in [-10, 10]$, and $x^{ca} \in \{a, b\}$. It is

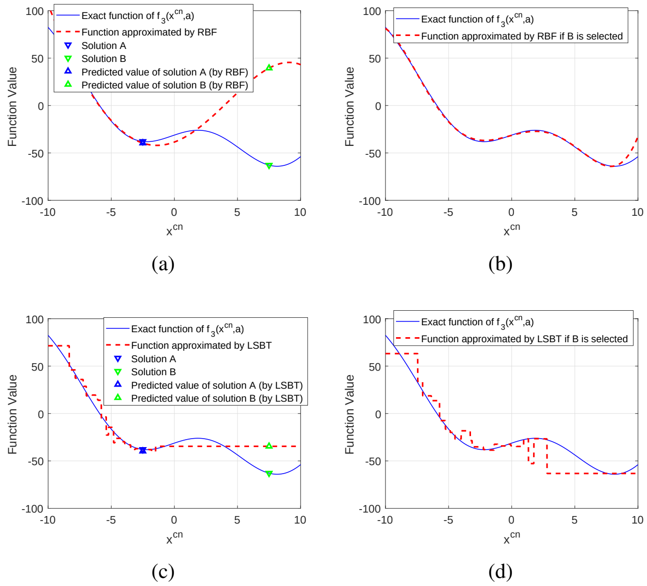

Fig. 4. Approximate $f_3(x^{cn},x^{ca})$ by RBF and LSBT: (a) The landscapes of the exact function of $f_3(x^{cn},x^{ca})$ and the function approximated by RBF when $x^{ca}=a$. (b) The landscapes of the exact function of $f_3(x^{cn},x^{ca})$ and the function approximated by RBF when $x^{ca}=a$ if **B** is selected. (c) The landscapes of the exact function of $f_3(x^{cn},x^{ca})$ and the function approximated by LSBT when $x^{ca}=a$. (d) The landscapes of the exact function of $f_3(x^{cn},x^{ca})$ and the function approximated by LSBT when $x^{ca}=a$ if **B** is selected.

approximated by RBF and LSBT, respectively. For convenience, we only exhibit the landscapes of the exact function of $f_3(x^{cn},x^{ca})$ and the function approximated by RBF/LSBT when $x^{ca}=a$. Our aim is to select a better solution from solutions **A** and **B**. Note that the original objective function value of **B** is better than that of **A**. However, both RBF and LSBT provide a better predicted value for **A**, which means **A** will be selected. In contrast, if the random selection is employed, we still have a chance to select **B**, thus improving the accuracy of the surrogate models as shown in Fig. 4(b) and Fig. 4(d) .

C. Surrogate-Assisted Local Search

Consider that the local search strategy can effectively improve the convergence speed [30], [49], in this paper, we also design surrogate-assisted local search to accelerate the convergence. We incorporate RBF into SQP to further optimize the continuous variables of the current best solution in $\mathbb{DB}$ (denoted as $\mathbf{x}_{best}=[\mathbf{x}_{best}^{cn},\mathbf{x}_{best}^{ca}]$). The implementation of the surrogate-assisted local search is explained as follows.

Firstly, we count the number of the solutions that have the same categorical variables with $\mathbf{x}_{best}$ (denoted as N_{ls}). If N_{ls} is bigger than a threshold (denoted as N_{min}), these solutions are used to construct a RBF surrogate model (denoted as $\hat{f}_{sub}$) for only continuous variables. Then, SQP is used to solve the following optimization problem:

$$\begin{aligned} min:\ & \hat{f}_{sub}(\mathbf{x}^{cn}) \\ s.t.\ & L_i^{cn}\le x_i^{cn}\le U_i^{cn} \end{aligned} \tag{18}$$

Based on the continuous vector obtained by SQP (denoted as $\mathbf{x}_{ls}^{cn}$), a new solution is produced: $\mathbf{x}_{ls}=[\mathbf{x}_{ls}^{cn},\mathbf{x}_{best}^{ca}]$. Finally, $\mathbf{x}_{ls}$ is reserved into $\mathbb{X}_{ls}$.

Next, we would like to give two comments on the surrogate-assisted local search:

- Commonly, it is hard to guarantee the accuracy of RBF if N_{ls} is too small. Therefore, a threshold is adopted to ensure the size of the data points.
- Since it is almost impossible that many solutions have the same continuous vector, it is very hard to construct a LSBT surrogate model for only categorical variables. Therefore, we do not improve the categorical variables.

V. Experimental Studies

A. Test Problems and Parameters Settings

1) Artificial Test Problems: The first set of test problems contains 30 artificial test problems (i.e., F1-F30). They are originated from five classical continuous functions: Sphere function, Rastrigin function, Alckey function, Elliposoid function, and Griewank function. Their characteristics are listed in Table S-XV of the supplementary files. According to their characteristics, we roughly classify them into three types:

- Type 1: most of the variables are continuous variables
- Type 2: most of the variables are categorical variables
- Type 3: the number of continuous variables is similar to that of categorical variables

Obviously, F1-F10 are type-1 artificial test problems, F11-F20 are type-2 artificial test problems, and F21-F30 are type-3 artificial test problems.

2) Capacitated Facility Location Problems: Six capacitated facility location problems (i.e., CFLP1-CFLP6) are constructed in this paper. The capacitated facility location problems can be formulated as below:

$$\begin{aligned} min:\ & f(\mathbf{x},\mathbf{y})=\sum_{i\in I}F_{i,y_i}y_i+\sum_{i\in I}\sum_{j\in J}Q_{i,j}x_{i,j} \\ & \sum_{i\in I}x_{i,j}=D_j, i\in I, j\in J \\ & \sum_{j\in J}x_{i,j}\le C_{y_i}y_i, i\in I, j\in J \\ & y_i\in S, i\in I \\ & x_{i,j}\ge 0, i\in I, j\in J \end{aligned}$$

where $I=\{1,\cdots,m\}$ represents a set of potential facility sites, $J=\{1,\cdots,n\}$ represents a set of customers, $S=\{0,\cdots,s\}$ represents a set of facility types, $C_r(r\in S)$ represents the capacity of the rth type of facility, $F_{i,r}$ represents the cost of operating the rth type of facility at site i, D_j represents the total demand of the jth customer, $Q_{i,j}$ represents the cost of serving a unit of demand for the jth customer from the ith facility, $x_{i,j}$ denotes the jth customer's demand from the ith facility, and y_i represents which type of facility is operated at site i (if $y_i=0$, no facility will be operated at site i).

3) Dubins Traveling Salesperson Problems: We also construct six Dubins traveling salesperson problems (i.e., DTSP1-DTSP6) in this paper. The Dubins travelling salesperson problems are related to the motion planning and task assignment

TABLE I
PARAMETER SETTINGS OF MISACO

Parameter	Value
Size of $\mathbb{OP}$: M	100
Influence of the best-quality solutions in ACO_{MV}: q	0.05099
Width of the search in ACO_{MV}: ξ	0.6795
Archive size in ACO_{MV}: K	60
Maximum number of function evolutions: $MaxFEs$	600
Threshold in the surrogate-assisted local search: N_{min}	$5*n_1$

for uninhabited vehicles. They can be formulated as follows:

$$
\begin{aligned}
min\ D(\mathbf{r},\mathbf{x}) = & \sum_{i=1}^{n-1} d(x_{r_i}, x_{r_{i+1}}) + d(x_{r_n}, x_{r_1}) \\
s.t.\ & r_i \neq r_j, \text{if } i \neq j \\
& r_i \in \{1,\cdots,n\} \\
& 0 \leq x_i \leq 2\pi \\
& i \in \{1,\ldots,n\} \\
& j \in \{1,\ldots,n\}
\end{aligned}
$$

where $\mathbf{r} = (r_1,\cdots,r_n)$ represents the sequence of waypoints needed to pass through, $r_i \in \{1,\ldots,n\}$ represents the ith waypoint, n represents the number of waypoints, $\mathbf{x} = \{x_1,\cdots,x_n\}$ represents the heading of the uninhabited vehicle at the ith waypoint, and $d(\cdot,\cdot)$ represents the shortest Dubins path between two waypoints. For $d(\cdot,\cdot)$, the shortest Dubins path between two waypoints must be one of the following six patterns: {RSL, LSR, RSR, LSL, RLR, LRL}, in which L, R, and S represent turning left with the minimal turning radius, turning right with the minimal turning radius, and moving along a straight line, respectively.

The details of these three sets of test problems are given in Section S-VI of the supplementary file. For each test problem, 20 independent runs were implemented. The parameter settings of MiSACO are listed in Table I. The settings of q and M were consistent with the original paper [10]. For each test problem, 20 independent runs were implemented.

To evaluate the performance of different algorithms, the following two indicators were calculated:

- *AOFV*: The average objective function value of the best solutions provided by an algorithm over 20 independent runs.
- *ASFEs*: The average FEs consumed by an algorithm to successfully obtain the optimal solution of a test problem over 20 independent runs. Note that, a run is considered as successful if the following condition is satisfied: $|f(\mathbf{x}_{best}) - f(\mathbf{x}^*)| \leq 1$, where $\mathbf{x}^*$ is the best known solution and $\mathbf{x}_{best}$ is the best solution provided by an algorithm. For an unsuccessful run, its consumed FEs was set to $MaxFEs$.

AOFV and *ASFEs* measure the convergence accuracy and efficiency of an algorithm, respectively. Since the optimal solutions of CFLP1-CFLP6 and DTSP1-DTSP6 are unknown, for these 12 problems, we did not calculate their *ASFEs* values. In the experimental studies, the Wilcoxon's rank-sum test at a 0.05 significance level was implemented between MiSACO and each of its competitors to test the statistical significance. In the following tables, "+", "−", and "≈" denote that MiSACO performs better than, worse than, and similar to its competitor, respectively.

B. Comparison with ACO_{MV}

In essence, MiSACO is an algorithm which combines surrogate models with ACO_{MV} for solving EOPCCVs. One may be interested in the performance difference between MiSACO and ACO_{MV}. To this end, we compared MiSACO with ACO_{MV}. To clearly exhibit their performance difference, we also calculated a performance metric called acceleration rate based on their *ASFEs* values:

$$AR = \frac{ASFEs_{ACO_{MV}} - ASFEs_{MiSACO}}{ASFEs_{ACO_{MV}}} \times 100\% \tag{19}$$

where $ASFEs_{ACO_{MV}}$ and $ASFEs_{MiSACO}$ are the *ASFEs* values of ACO_{MV} and MiSACO, respectively. Note that, if any of these two algorithms fails to find any optimal solution over 20 independent runs, the *ASFEs* value will be equal to *MaxFEs*. Under this condition, it is meaningless to calculate the acceleration rate. When this happens, the corresponding *AR* value is denoted as "*NA*". All the results are exhibited in Table II, Table III, Table S-I of the supplementary file, and Table S-II of the supplementary file.

The detailed discussions about the results are given as follows.

1) Results on the Artificial Test Problems:

- In terms of *AOFV*, it can be observed from Table II that MiSACO can obtain better values than ACO_{MV} on all the 30 artificial test problems. Since F2, F7, F12, F17, F22, and F27 originate from Rastrigin function, which is a function with a complex and multimodal landscape, it is very likely to build an inaccurate surrogate model. Therefore, MiSACO cannot find solutions with high accuracies when solving these six artificial test problems. However, MiSACO can still provide smaller *AOFV* values than ACO_{MV} on them. From the Wilcoxon's rank-sum test, MiSACO surpasses ACO_{MV} on all the 30 artificial test problems in terms of *AOFV*.
- As far as *ASFEs* is concerned, we can observe from Table II that the values provided by MiSACO are better than those resulting from ACO_{MV} on all the 30 artificial test problems except F2, F7, F12, F17, F22, and F27. When solving these six artificial test problems, both MiSACO and ACO_{MV} cannot successfully obtain the optimal solutions in any run. Therefore, their *ASFEs* values are equal to *MaxFEs*. According to the Wilcoxon's rank-sum test, MiSACO beats ACO_{MV} on 24 artificial test problems in terms of *ASFEs*. However, ACO_{MV} cannot outperform MiSACO on any artificial test problem.
- From Table III, MiSACO converges at least 30% faster than ACO_{MV} toward the optimal solutions on all the 30 artificial test problems except F2, F7, F12, F16, F17, F18, F20, F22, F27, and F28. On average, MiSACO reduces 43.98% *ASFEs* to reach the optimal solutions against ACO_{MV}. Specifically, MiSACO saves 52.49%, 34.64%, and 44.82% *ASFEs* on solving type-1, type-2, and type-3 artificial test problems, respectively.

TABLE II
RESULTS OF ACO$_{MV}$ AND MISACO OVER 20 INDEPENDENT RUNS ON THE 30 ARTIFICIAL TEST PROBLEMS. THE WILCOXON'S RANK-SUM TEST AT A 0.05 SIGNIFICANCE LEVEL WAS PERFORMED BETWEEN ACO$_{MV}$ AND MISACO.

	Problem	ACO$_{MV}$		MiSACO	ACO$_{MV}$		MiSACO
		AOFV ± *Std Dev*		*AOFV* ± *Std Dev*	*ASFEs* ± *Std Dev*		*ASFEs* ± *Std Dev*
Type 1	F1	4.12E+00 ± 2.61E+00	+	**6.21E−08 ± 2.24E−08**	600.00 ± 0.00	+	**249.95 ± 36.49**
	F2	6.04E+01 ± 1.22E+01	+	**2.59E+01 ± 1.19E+01**	**600.00 ± 0.00**	≈	**600.00 ± 0.00**
	F3	2.04E+00 ± 6.16E−01	+	**3.04E−01 ± 7.50E−01**	600.00 ± 0.00	+	**375.85 ± 101.67**
	F4	2.63E−01 ± 2.13E−01	+	**4.51E−09 ± 2.25E−09**	457.90 ± 71.55	+	**159.10 ± 25.60**
	F5	1.01E+00 ± 1.26E−01	+	**2.39E−01 ± 2.30E−01**	587.95 ± 30.73	+	**208.90 ± 37.03**
	F6	6.43E+00 ± 6.28E+00	+	**5.74E−08 ± 2.50E−08**	593.60 ± 19.08	+	**269.40 ± 56.18**
	F7	5.53E+01 ± 7.09E+00	+	**2.40E+01 ± 1.22E+01**	**600.00 ± 0.00**	≈	**600.00 ± 0.00**
	F8	1.79E+00 ± 7.11E−01	+	**7.27E−02 ± 2.83E−01**	597.55 ± 8.89	+	**391.60 ± 63.22**
	F9	3.38E−01 ± 2.98E−01	+	**2.59E−03 ± 8.51E−03**	469.25 ± 77.56	+	**231.15 ± 65.29**
	F10	1.08E+00 ± 1.10E−01	+	**2.75E−01 ± 3.01E−01**	595.60 ± 16.49	+	**270.20 ± 123.87**
Type 2	F11	2.39E+00 ± 4.59E+00	+	**9.43E−08 ± 1.06E−07**	533.55 ± 81.34	+	**285.75 ± 73.21**
	F12	5.19E+01 ± 2.00E+01	+	**4.71E+01 ± 1.73E+01**	**600.00 ± 0.00**	≈	**600.00 ± 0.00**
	F13	1.15E+00 ± 9.63E−01	+	**4.19E−01 ± 1.29E+00**	546.75 ± 96.96	+	**307.25 ± 120.94**
	F14	9.10E−03 ± 8.17E−03	+	**2.04E−09 ± 2.03E−09**	354.25 ± 80.54	+	**195.45 ± 39.24**
	F15	7.80E−01 ± 3.02E−01	+	**2.82E−07 ± 3.27E−07**	503.85 ± 86.41	+	**244.50 ± 44.50**
	F16	2.82E+01 ± 4.67E+01	+	**1.27E+00 ± 5.66E+00**	564.75 ± 62.57	+	**409.65 ± 100.36**
	F17	6.36E+01 ± 1.30E+01	+	**4.50E+01 ± 1.61E+01**	**600.00 ± 0.00**	≈	**600.00 ± 0.00**
	F18	1.71E+00 ± 1.43E+00	+	**1.55E+00 ± 1.96E+00**	553.00 ± 70.48	+	**538.25 ± 93.69**
	F19	8.71E−01 ± 6.51E−01	+	**2.71E−01 ± 7.16E−01**	521.60 ± 85.39	+	**356.45 ± 122.27**
	F20	1.65E+00 ± 1.08E+00	+	**1.05E−01 ± 3.23E−01**	564.10 ± 64.50	+	**401.70 ± 109.14**
Type 3	F21	7.08E+00 ± 4.92E+00	+	**7.60E−08 ± 5.64E−08**	599.75 ± 1.12	+	**279.55 ± 36.60**
	F22	6.20E+01 ± 1.33E+01	+	**4.78E+01 ± 1.32E+01**	**600.00 ± 0.00**	≈	**600.00 ± 0.00**
	F23	2.43E+00 ± 8.52E−01	+	**1.52E−01 ± 6.77E−01**	597.35 ± 11.85	+	**363.15 ± 76.32**
	F24	3.09E−01 ± 3.10E−01	+	**9.90E−07 ± 4.42E−06**	444.45 ± 88.68	+	**185.15 ± 37.79**
	F25	1.07E+00 ± 6.24E−02	+	**1.24E−01 ± 2.34E−01**	598.50 ± 6.71	+	**229.45 ± 32.75**
	F26	2.76E+01 ± 7.30E+01	+	**9.70E−08 ± 8.28E−08**	600.00 ± 0.00	+	**330.80 ± 62.07**
	F27	6.46E+01 ± 8.30E+00	+	**4.15E+01 ± 1.78E+01**	**600.00 ± 0.00**	≈	**600.00 ± 0.00**
	F28	1.77E+00 ± 1.07E+00	+	**9.76E−01 ± 1.46E+00**	589.85 ± 18.96	+	**475.15 ± 117.83**
	F29	3.74E−01 ± 3.27E−01	+	**3.74E−02 ± 1.44E−01**	459.60 ± 84.79	+	**273.10 ± 83.76**
	F30	1.16E+00 ± 2.41E−01	+	**4.92E−01 ± 6.18E−01**	600.00 ± 0.00	+	**353.50 ± 118.11**
+/−/≈		30/0/0			24/0/6		

TABLE III
ACCELERATION RATE OF MISACO AGAINST ACO$_{MV}$ ON THE 30 ARTIFICIAL TEST PROBLEMS.

Type 1		Type 2		Type 3	
Problem	*AR*	Problem	*AR*	Problem	*AR*
F1	58.34%	F11	46.44%	F21	53.39%
F2	*NA*	F12	*NA*	F22	*NA*
F3	37.36%	F13	43.80%	F23	39.21%
F4	65.25%	F14	44.83%	F24	58.34%
F5	64.47%	F15	51.47%	F25	61.66%
F6	54.62%	F16	27.46%	F26	44.87%
F7	*NA*	F17	*NA*	F27	*NA*
F8	34.47%	F18	2.67%	F28	19.45%
F9	50.74%	F19	31.66%	F29	40.58%
F10	54.63%	F20	28.79%	F30	41.08%
				Average *AR*	43.98%

2) Results on the Capacitated Facility Location Problems:

- From Table S-I, for all the six capacitated facility location problems, MiSACO obtains better *AOFV* values than ACO$_{MV}$. According to the Wilcoxon's rank-sum test, MiSACO beats ACO$_{MV}$ on all these six problems.

3) Results on the Dubins Traveling Salesman Problems:

- From Table S-II, for all the six Dubins traveling salesman problems, MiSACO provides better *AOFV* values. According to the Wilcoxon's rank-sum test, MiSACO performs better than ACO$_{MV}$ on all these six problems.

From the above discussion, it can be concluded that the proposed multi-surrogate-assisted selection and surrogate-assisted local search can significantly enhance the convergence accuracy and efficiency of ACO$_{MV}$.

C. Comparison with Other State-of-the-Art SAEAs

To further test the performance of MiSACO, we compared it with CAL-SAPSO [49], EGO-Hamming [66], EGO-Gower [63], and BOA-RF [67]. CAL-SAPSO is a SAEA for continuous EOPs. To make it have the capability to deal with categorical variables in EOPCCVs, we encoded each element in the candidate categorical set of each categorical variable into an ordered integer. The rounding operator was used in the optimization process. EGO-Hamming and EGO-Gower are two extended versions of efficient global optimization (EGO) [68]. By redefining the distance between two different categorical vectors, EGO-Hamming and EGO-Gower are able to solve EOPCCVs directly. In these two algorithms, Hamming distance and Gower distance were employed to measure the difference between two different categorical vectors, respectively. BOA-RF is a variant of Bayesian optimization [69]. Inspired by the sequential model-based algorithm configuration [67], we employed RF as the surrogate model and used the expected improvement as the acquisition function in BOA-RF.

The results provided by CAL-SAPSO, EGO-Hamming, EGO-Gower, BOA-RF, and MiSACO are recorded in Table S-III–Table S-VI of the supplementary file. The detailed discussions are given below:

1) Results on the Artificial Test Problems:

- From Table S-III, MiSACO can provide smaller *AOFV* values than CAL-SAPSO on all the 30 artificial test problems. Moreover, each *AOFV* value of MiSACO is at least one order smaller than the corresponding *AOFV* value of CAL-SAPSO on all the 30 artificial test problems except F2, F7, F12, F17, F22, and F27. From Table S-IV, MiSACO provides smaller *ASFEs* values than CAL-SAPSO on 24 artificial test problems (F1, F3-F6, F8-F11, F13-F16, F18-F21, F23-F26, and F28-F30). Thus, MiSACO can find the optimal solutions faster on these 24 artificial test problems. According to the Wilcoxon's rank-sum test, MiSACO surpasses CAL-SAPSO on 29 artificial test problems in terms of *AOFV* and 24 artificial test problems in terms of *ASFEs*, respectively.
- Compared with EGO-Hamming, MiSACO provides better *AOFV* values and better *ASFEs* values on 29 artificial test problems (F1-F11 and F13-F30) and 24 artificial test problems (F1, F3-F6, F8-F11, F13-F16, F18-F21, F23-F26, and F28-F30), respectively. According to the Wilcoxon's rank-sum test, MiSACO has an edge over EGO-Hamming on 27 artificial test problems in terms of *AOFV* and 24 artificial test problems in terms of *ASFEs*, respectively. However, EGO-Hamming cannot beat MiSACO on any artificial test problem in terms of any performance indicator.
- Compared with MiSACO, EGO-Gower provides worse *AOFV* values on 27 artificial test problems and better *AOFV* values on only three artificial test problems (F12, F17, and F22). With respect to *ASFEs*, EGO-Gower is worse than MiSACO on 24 artificial test problems (F1, F3-F6, F8-F11, F13-F16, F18-F21, F23-F26, and F28-F30), and cannot provide any better value on any artificial test problem. According to the Wilcoxon's rank-sum test, MiSACO beats EGO-Gower on 26 artificial test problems in terms of *AOFV* and 24 artificial test problems in terms of *ASFEs*, respectively.
- BOA-RF provides worse *AOFV* and *ASFEs* values than MiSACO on 30 and 24 (i.e., F1, F3-F6, F8-F11, F13-F16, F18-F21, F23-F26, and F27-F30) artificial test problems, respectively. Meanwhile, with respect to both *AOFV* and *ASFEs*, MiSACO does not provide any worse value than BOA-RF on any artificial test problem. According to the Wilcoxon's rank-sum test, MiSACO beats BOA-RF on 30 artificial test problems in terms of *AOFV* and 24 artificial test problems in terms of *ASFEs*, respectively. Moreover, MiSACO does not lose on any artificial test problem.

2) Results on the Capacitated Facility Location Problems:

- From Table S-V, MiSACO obtains better *AOFV* values than CAL-SAPSO, EGO-Hamming, and BOA-RF on all the six capacitated facility location problems. Moreover, MiSACO is better than EGO-Gower on five problems. According to the Wilcoxon's rank-sum test, MiSACO beats CAL-SAPSO, EGO-Hamming, EGO-Gower, and BOA-RF on six, six, three, and five problems, respectively.

TABLE IV
RESULTS ABOUT THE STRUCTURES OF THE STIFFENED PLATES REPORTED IN [70] AND OBTAINED BY EGO-GOWER, GA, AND MISACO.

Status	Reported in [70]	EGO-Gower	GA	MiSACO
D	8.020 mm	6.828 mm	7.880 mm	6.489 mm
$MASS$	0.124 kg	0.124 kg	0.123 kg	0.124 kg

3) Results on the Dubins Traveling Salesman Problems:

- From Table S-VI, MiSACO is better than CAL-SAPSO, EGO-Hamming, EGO-Gower, and BOA-RF on all the six Dubins traveling salesman problems in terms of *AOFV*. According to the Wilcoxon's rank-sum test, MiSACO beats the four competitors on all the six problems.

The above results demonstrate that, overall, the performance of MiSACO is better than that of the four state-of-the-art competitors in terms of both *AOFV* and *ASFEs*. The superiority of MiSACO against the four competitors can be attributed to the fact that these four competitors mainly extend surrogate models for continuous functions, thus having limited capabilities to cope with EOPCCVs.

VI. REAL-WORLD APPLICATIONS

In this section, MiSACO was applied to solve two EOPCCVs in the real world, i.e., the topographical design of stiffened plates against blast loading, and the lightweight and crashworthiness design for the side body of an automobile.

A. Topographical Design of Stiffened Plates Against Blast Loading

At present, the explosion caused by accidents and terrorist attacks has attracted more and more attention. In order to protect the personnel and facilities from explosion damage, research on blast-resistant structures is of great significance. As a kind of structure that can effectively deal with blast loading, stiffened plates have been widely studied [71], [72]. Recently, Liu *et al.* [70] designed the topographical structures of two new kinds of stiffened plates. Based on their research work, we tried to assign different materials for different stiffeners, thus further improving the structural resistance of the stiffened plates against blast loading.

As shown in Fig. S-1(a) in Section S-V of the supplementary file, one kind of the stiffened plate in [70] is considered in this paper. The front plate of the stiffened plate is a square shape with $L \times L = 250mm \times 250mm$, and its thickness is 1 mm. The height of each stiffener is $H = 10$ mm. Fig. S-1(b) depicts the variable distribution of this structure. We can observe that this structure is 1/8 symmetric; hence, the thicknesses and materials of the 13 stiffeners are considered as the design variables, i.e., $x_1^{thick}, \ldots, x_{13}^{thick}$ and $x_1^{mat}, \ldots, x_{13}^{mat}$. Same with [70], the maximum deflection of the center point of the plate (denoted as D) is employed to assess the structural resistance, and the mass of the plate (denoted as $MASS$) is constrained within a certain value (denoted as M^*). Thus, this

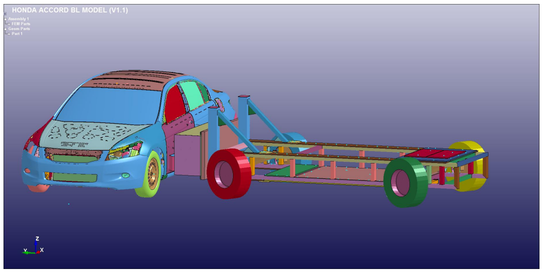

Fig. 5. Side crash FEA model considered in this paper. In this model, Honda Accord was employed as the baseline.

design problem can be described as follows:

$$\begin{aligned} & min: D(\mathbf{x}^{thick}, \mathbf{x}^{mat}) \\ & s.t.\ MASS(\mathbf{x}^{thick}, \mathbf{x}^{mat}) \leq M^* \\ & \qquad \mathbf{x}^{thick} = (x_1^{thick}, \ldots, x_{13}^{thick}) \\ & \qquad \mathbf{x}^{mat} = (x_1^{mat}, \ldots, x_{13}^{mat}) \\ & \qquad x_1^{thick}, \ldots, x_{13}^{thick} \in [0, 2] \\ & \qquad x_1^{mat}, \ldots, x_{13}^{mat} \in \{MAT1, \ldots, MAT5\} \end{aligned} \tag{20}$$

where $MAT1, \ldots, MAT5$ represent the five categories of steel: mild steel, IF300/420, DP350/600, IF260/400, and DP500/800. The value of M^* was set to $0.09705kg$.

MiSACO was used to optimize the structure of the stiffened plate, and $MaxFEs$ was set to 800. For comparison, we also employed EGO-Gower and GA to optimize the structure. Since the value of $MASS$ can be calculated directly according to the density of materials and the thicknesses of stiffeners, the constraint in (20) is not an expensive function. Thus, in these three algorithms, the feasibility rule [73][2] was used to deal with the constraint. Table IV summarizes the values of D and $MASS$ of the structures reported in [70] and presented by the three algorithms. Moreover, the topographical structures of the four corresponding stiffened plates are shown in Fig. S-2 in Section S-V of the supplementary file.

From Table IV, compared with the structure reported in [70], the structure provided by MiSACO improves the D value by 19.09%. Meanwhile, these two structures have similar structure mass. This indicates that assigning different materials for different stiffeners can effectively improve the structural resistance. Compared with the structures provided by EGO-Gower and GA, the structure resulting from MiSACO has the best D value. Moreover, Fig. S-3 plots the convergence curves derived from EGO-Gower, GA, and MiSACO. It can be observed from Fig. S-3 that MiSACO can provide the best convergence performance.

B. *Lightweight and Crashworthiness Design for the Side Body of An Automobile*

In the field of automotive engineering, it is desirable to design an automobile body with low mass and high crashworthiness, thus reducing the fuel consumption and improving the safety of the automobile. In this paper, we focus on the design of the side body of an automobile.

[2] The feasibility rule compares two solutions as follows: 1) Between two infeasible solutions, the one with smaller degree of constraint violation is preferred, 2) If one solution is infeasible and the other is feasible, the feasible one is preferred, and 3) Between two feasible solutions, the one with a better objective function value is preferred.

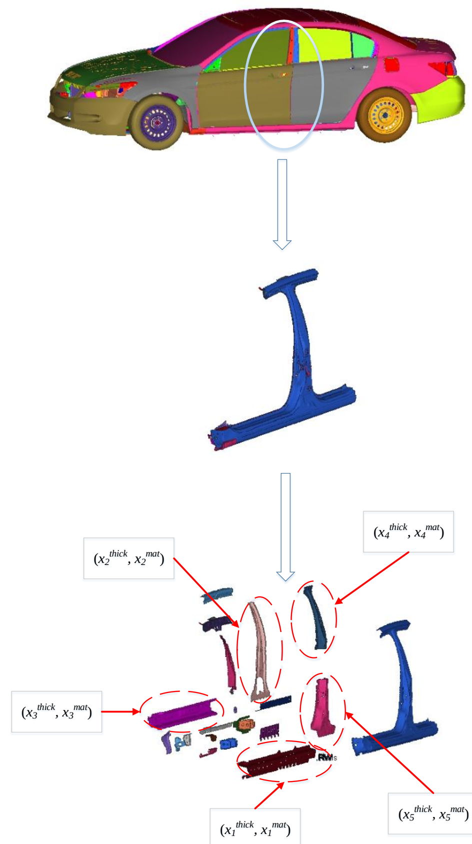

Fig. 6. Five thin-walled plate parts which need to be optimized

The side crash FEA model is shown in Fig. 5. The FEA model was established according to Honda Accord, and the details about this model can be found from the technical report provided by Singh *et al.* [15]. We selected five thin-walled plate parts from the side body of the automobile, and tried to redesign their thicknesses and materials. These five thin-walled plate parts are shown in Fig. 6. As it is very time-consuming to perform a side crash simulation by using the FEA model described in Fig. 5, in this paper we used a simplified FEA model, as shown in Fig. S-4. It contains B-pillar and a part of the side door of the original automobile. On our computer, this simplified FEA model took about 50 minutes to execute a run.

In this paper, the following three indicators were used to assess the structure:

- *The maximum invasion at the middle of B-pillar*: Reducing the maximum invasion at the middle of B-pillar can improve the safety of passengers in the automobile. We denote the maximum invasion at the middle of B-pillar as FI.
- *The maximum invasion velocity at the middle of B-pillar*: Commonly, a small maximum invasion velocity at the

TABLE V
RESULTS OF THE ORIGINAL DESIGN AND THE DESIGN PROVIDED BY MISACO.

Status	Original Design	MiSACO
FI	240.04 *mm*	177.26 *mm*
FV	9674.7 *mm/s*	9659.7 *mm/s*
$MASS$	0.0252 *t*	0.0251 *t*

middle of B-pillar can reduce the probability of passenger injury. In this paper, the maximum invasion velocity at the middle of B-pillar is denoted as FV.

- *The mass of the side body*: This indicator is to evaluate the lightweight level of the automobile. We denote this indicator as $MASS$.

According to the simplified FEA model, the values of FI, FV, and $MASS$ of the original design are 240.04*mm*, 9674.7*mm/s*, and 0.0252*t*, respectively[3]. Our aim is to reduce the FI value without increasing the FV and $MASS$ values. Overall, this design can be described as follows:

$$\begin{aligned} min:\ & FI(\mathbf{x}^{thick},\mathbf{x}^{mat}) \\ s.t.\ & FV(\mathbf{x}^{thick},\mathbf{x}^{mat}) \leq 9674.7 \\ & MASS(\mathbf{x}^{thick},\mathbf{x}^{mat}) \leq 0.0252 \\ & \mathbf{x}^{thick} = (x_1^{thick},\ldots,x_5^{thick}) \\ & \mathbf{x}^{mat} = (x_1^{mat},\ldots,x_5^{mat}) \\ & x_1^{thick},\ldots,x_5^{thick} \in [0.2,2] \\ & x_1^{mat},\ldots,x_5^{mat} \in \{MAT1,\ldots,MAT6\} \end{aligned} \tag{21}$$

where $x_1^{thick},\ldots,x_5^{thick}$ are the thickness variables of the five thin-walled plate parts, and $x_1^{mat},\ldots,x_5^{mat}$ are the material variables of the five thin-walled plate parts. There are six kinds of materials for each thin-walled plate part: $MAT1,\ldots,MAT6$, which represent six kinds of steel: DP350/600, DP500/800, HSLA350/450, IF140/270, IF260/410, and IF300/420.

We consumed 500 FEs to optimize the side body by using MiSACO. The whole optimization process took about $(50*500)/(60*24) \approx 17.36$ days. The feasibility rule [73] was used to deal with the constraints in (21). For $FI(\mathbf{x}^{thick},\mathbf{x}^{mat})$ and $FV(\mathbf{x}^{thick},\mathbf{x}^{mat})$, the surrogate models were constructed independently. Since $MASS(\mathbf{x}^{thick},\mathbf{x}^{mat})$ can be calculated directly, we did not establish any surrogate model for it. The values of FI, FV, and $MASS$ of the original design and MiSACO are listed in Table V. Compared with the original design, MiSACO improves the FI value by 26.16%. At the same time, the FV and $MASS$ values of the original design are similar to those provided by MiSACO. It means that the crashworthiness of the side body of the automobile can be greatly enhanced without adding the mass of the structure. Thus, the safety of the automobile can be significantly improved.

The above experiments reveal that MiSACO could be an effective tool to solve EOPCCVs in the real world.

[3]In our experiment, LS-DYNA was employed as the simulation solver, and its version was “ls971s R4.2”. The whole simulation process was executed in “Windows 7 64-bit Ultimate”.

VII. CONCLUSION

Many real-world applications can be modeled as EOPCCVs. However, few attempts have been made on solving EOPCCVs. In this paper, we proposed a multi-surrogate-assisted ACO, called MiSACO, to solve EOPCCVs. MiSACO contained two main strategies: the multi-surrogate-assisted selection and the surrogate-assisted local search. In the former, three selection operators (i.e., the RBF-based selection, the LSBT-based selection, and the random selection) were employed. The aim of them is to help MiSACO to deal with different types of EOPCCVs robustly and prevent MiSACO from being misled by inaccurate surrogate models. In the latter, we focused on improving the quality of the current best solution by further optimizing its continuous variables. This strategy was implemented by constructing a RBF for only continuous variables and optimizing this RBF by using SQP. From the comparative studies on the three sets of test problems, the effectiveness of MiSACO was verified. We also applied MiSACO to solve two EOPCCVs in the real world: the topographical design of stiffened plates against blast loading, and the lightweight and crashworthiness design for the side body of an automobile. The results showed that MiSACO can effectively solve them.

In the future, we will apply MiSACO to solve large-scale EOPCCVs (e.g., increase the number of categorical variables and/or the size of candidate categorical sets). When the scale of an EOPCCV becomes larger, it is not easy to construct an accurate surrogate model. Some techniques such as dimension reduction methods will be further incorporated into MiSACO to deal with large-scale EOPCCVs. In addition, in the surrogate-assisted local search, if the best solution does not change for several generations, it will be optimized repeatedly; thus, the computational resources may be wasted. In the future, we will try to address this limitation.

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Supplementary File for "Multi-Surrogate-Assisted Ant Colony Optimization for Expensive Optimization Problems with Continuous and Categorical Variables"

S-I. Proof of Propositions

A. Proof of Proposition 1

The upper bound of RBF's predicted error can be estimated as:

$$\begin{aligned}
&|f(\mathbf{x}^{cn},\mathbf{x}^{ca})-\hat{f}_{RBF}(\mathbf{x}^{cn},\mathbf{x}^{ca})|\\
=&|f(\mathbf{x}^{cn},\mathbf{x}^{ca})-f(\mathbf{x}^{cn}_*,\mathbf{x}^{ca}_*)+\hat{f}_{RBF}(\mathbf{x}^{cn}_*,\mathbf{x}^{ca}_*)-\hat{f}_{RBF}(\mathbf{x}^{cn},\mathbf{x}^{ca})|\\
\le&|f(\mathbf{x}^{cn},\mathbf{x}^{ca})-f(\mathbf{x}^{cn}_*,\mathbf{x}^{ca}_*)|+|\hat{f}_{RBF}(\mathbf{x}^{cn}_*,\mathbf{x}^{ca}_*)-\hat{f}_{RBF}(\mathbf{x}^{cn},\mathbf{x}^{ca})|\\
\le&(L_f+L_r)\cdot dis((\mathbf{x}^{cn},\mathbf{x}^{ca}),(\mathbf{x}^{cn}_*,\mathbf{x}^{ca}_*))
\end{aligned}$$

where the first equality holds by Assumption 2, the second inequality is the triangle inequality, and the third inequality holds by Assumption 1.

Similarly, the lower bound of RBF's predicted error can be estimated as:

$$\begin{aligned}
&|f(\mathbf{x}^{cn},\mathbf{x}^{ca})-\hat{f}_{RBF}(\mathbf{x}^{cn},\mathbf{x}^{ca})|\\
=&|f(\mathbf{x}^{cn},\mathbf{x}^{ca})-f(\mathbf{x}^{cn}_*,\mathbf{x}^{ca}_*)+\hat{f}_{RBF}(\mathbf{x}^{cn}_*,\mathbf{x}^{ca}_*)-\hat{f}_{RBF}(\mathbf{x}^{cn},\mathbf{x}^{ca})|\\
\ge&\big||f(\mathbf{x}^{cn},\mathbf{x}^{ca})-f(\mathbf{x}^{cn}_*,\mathbf{x}^{ca}_*)|-|\hat{f}_{RBF}(\mathbf{x}^{cn}_*,\mathbf{x}^{ca}_*)-\hat{f}_{RBF}(\mathbf{x}^{cn},\mathbf{x}^{ca})|\big|
\end{aligned}$$

Let $F=\big||f(\mathbf{x}^{cn},\mathbf{x}^{ca})-f(\mathbf{x}^{cn}_*,\mathbf{x}^{ca}_*)|-|\hat{f}_{RBF}(\mathbf{x}^{cn}_*,\mathbf{x}^{ca}_*)-\hat{f}_{RBF}(\mathbf{x}^{cn},\mathbf{x}^{ca})|\big|$. According to Assumption 1, for F, we have:

$$0\le F\le\max(L_f-\frac{1}{L_r},L_r-\frac{1}{L_f})\cdot dis((\mathbf{x}^{cn},\mathbf{x}^{ca}),(\mathbf{x}^{cn}_*,\mathbf{x}^{ca}_*))$$

thus

$$|f(\mathbf{x}^{cn},\mathbf{x}^{ca})-\hat{f}_{RBF}(\mathbf{x}^{cn},\mathbf{x}^{ca})|\ge 0$$

B. Proof of Proposition 2

The upper bound of LSBT's predicted error can be estimated as:

$$\begin{aligned}
&|f(\mathbf{x}^{cn},\mathbf{x}^{ca})-\hat{f}_{LSBT}(\mathbf{x}^{cn},\mathbf{x}^{ca})|\\
=&|f(\mathbf{x}^{cn},\mathbf{x}^{ca})-f(\mathbf{x}^{cn}_*,\mathbf{x}^{ca}_*)+\hat{f}_{LSBT}(\mathbf{x}^{cn}_*,\mathbf{x}^{ca}_*)-\hat{f}_{LSBT}(\mathbf{x}^{cn},\mathbf{x}^{ca})|\\
=&|f(\mathbf{x}^{cn},\mathbf{x}^{ca})-f(\mathbf{x}^{cn}_*,\mathbf{x}^{ca}_*)|\\
\le&L_f\cdot dis((\mathbf{x}^{cn},\mathbf{x}^{ca}),(\mathbf{x}^{cn}_*,\mathbf{x}^{ca}_*))
\end{aligned}$$

where the first equality holds by Assumption 2, the second equality holds by Assumption 3, and the third inequality holds by Assumption 1.

Similarly, the lower bound of LSBT's predicted error can be estimated as:

$$|f(\mathbf{x}^{cn},\mathbf{x}^{ca})-\hat{f}_{LSBT}(\mathbf{x}^{cn},\mathbf{x}^{ca})|\ge\frac{1}{L_f}\cdot dis((\mathbf{x}^{cn},\mathbf{x}^{ca}),(\mathbf{x}^{cn}_*,\mathbf{x}^{ca}_*))$$

S-II. Discussions

A. Effectiveness of the Three Selection Operators

To investigate the effectiveness of the three selection operators (i.e., the RBF-based selection, the LSBT-based selection, and the random selection), we designed three variants of MiSACO, called MiSACO-LSBT-Rand, MiSACO-RBF-Rand, and MiSACO-LSBT-RBF. In MiSACO-LSBT-Rand, the RBF-based selection was not used; in MiSACO-RBF-Rand, the LSBT-based selection was removed; and in MiSACO-LSBT-RBF, the random selection was not employed. The *AOFV* and *ASFEs* values are summarized in Table S-VII and Table S-VIII of the supplementary file, respectively. The results are discussed in the following:

- MiSACO-LSBT-Rand cannot give any better *AOFV* or *ASFEs* value on the ten type-1 and ten type-3 artificial test problems. Thus, we can conclude that MiSACO-LSBT-Rand is not good at dealing with type-1 and type-3 artificial test problems. The reason may be that without the RBF-based selection, MiSACO-LSBT-Rand cannot solve EOPCCVs with many continuous variables well.
- MiSACO-RBF-Rand cannot get any better *AOFV* or *ASFEs* value on the ten type-2 and ten type-3 artificial test problems. Therefore, MiSACO-RBF-Rand performs poor on solving type-2 and type-3 artificial test problems. This is because only using the RBF-based selection and the random selection cannot help MiSACO-RBF-Rand to solve EOPCCVs with many categorical variables effectively.
- For eight out of the ten type-1 artificial test problems (F2, F3, and F5-F10), nine out of the ten type-2 artificial test problems (F11-F13 and F15-F20), and all of the ten type-3 artificial test problems, MiSACO can obtain better *AOFV* values than MiSACO-LSBT-RBF. Meanwhile, for four type-1 artificial test problems (F3, F5, F6, and F10), seven type-2 artificial test problems (F11, F13-F16, F19, and F20), and six type-3 artificial test problems (F21, F24, F26, and F28-F30), MiSACO can provide better *ASFEs* values. Obviously, for all of these three types of artificial test problems, the random selection is able to enhance the performance of the algorithm. This is because when the constructed surrogate models are inaccurate, the random selection has the potential to improve their accuracies.

From the above analysis, we can conclude that all of the three selection operators are indispensable.

B. Effectiveness of the Surrogate-Assisted Local Search

We would like to ascertain whether the surrogate-assisted local search can improve the convergence performance of MiSACO. To this end, additional experiments were conducted. A variant of MiSACO, called MiSACO-WoLocal, was devised. In MiSACO-WoLocal, the surrogate-assisted local search was removed. The *AOFV* and *ASFEs* values provided by MiSACO-WoLocal and MiSACO are given in Table S-IX of the supplementary file.

For *AOFV*, MiSACO can obtain better values on all the 30 test problems except F25 and F29. For *ASFEs*, MiSACO can provide better values on 23 test problems (F1, F3-F6, F8-F11, F13-F15, F18-F21, F23-F25, and F28-F30). According to the Wilcoxon's rank-sum test, MiSACO performs better than MiSACO-WoLocal on 20 test problems in terms of *AOFV* and nine test problems in terms of *ASFEs*, respectively. Based on the above results, it can be concluded that the surrogate-assisted local search is capable of enhancing the convergence performance of MiSACO.

C. Effectiveness of RBF in MiSACO

In MiSACO, RBF was used as the surrogate model for continuous functions. However, other popular techniques, such as Kriging, can also be employed. One may be interested in the influence of RBF and Kriging on the performance of MiSACO. To this end, a variant of MiSACO, called MiSACO-Kriging, was designed. The *AOFV* and *ASFEs* values and runtime provided by MiSACO-Kriging and MiSACO are recorded in Table S-X and Table S-XI of the supplementary file, respectively.

From Table S-X and Table S-XI, MiSACO-Kriging and MiSACO show similar performance in terms of both *AOFV* and *ASFEs*. However, the runtime consumed by MiSACO-Kriging is significantly longer than that of MiSACO. This is because the computational time complexity of Kriging is much higher than that of RBF. Therefore, we employed RBF as the surrogate model for continuous functions in this paper.

D. Effectiveness of LSBT in MiSACO

In MiSACO, LSBT was employed as the surrogate model with a tree structure. As RF is also a famous surrogate model with a tree structure, one may be interested in whether RF can be incorporated into MiSACO. To answer this question, a variant of MiSACO, called MiSACO-RF, was developed by replacing LSBT with RF. The *AOFV* and *ASFEs* values provided by MiSACO-RF and MiSACO are recorded in Table S-XII of the supplementary file.

From Table S-XII, in terms of *AOFV*, MiSACO is better than MiSACO-RF on 27 test problems (i.e., F1-F4, F6-F21, F23-F26, and F28-F30). As far as *ASFEs* is concerned, MiSACO obtains better values on 24 test problems (i.e., F1, F3-F6, F10, F11, F13-16, F19-F21, F23-F26, and F28-F30). According to the Wilcoxon's rank-sum test, MiSACO performs better than MiSACO-RF on 23 test problems in terms of *AOFV* and 20 test problems in terms of *ASFEs*, respectively. Therefore, in this paper, we employed LSBT as the surrogate model with a tree structure.

The superiority of MiSACO against MiSACO-RF may be attributed to the following reasons. When training each binary regression tree in a RF, only a certain numbers of solutions in $\mathbb{DB}$ are used. If the number of solutions in $\mathbb{DB}$ is small, fewer solutions will be used to train each binary regression tree in a RF. At this time, it is difficult to guarantee the accuracy of each binary regression tree in a RF. As a result, the trained RF is also difficult to provide an accurate prediction. In contrast, when training each binary regression tree in a LSBT, all of the solutions in $\mathbb{DB}$ are used. This makes it possible for LSBT to provide better predicted values than RF. Since the number of solutions in $\mathbb{DB}$ is always a small value, we prefer to employ LSBT in our algorithm.

E. Influence of Different Distances in RBF

In Section II, we have mentioned that, to handle EOPCCVs, the distance used in RBF should be redefined. However, we have also mentioned another distance in Section V, i.e., Gower distance. One may be interested in the performance of MiSACO if Gower distance is used in RBF. To investigate this, we designed a variant of MiSACO, called MiSACO-Gower. In MiSACO-Gower, Gower distance was employed in RBF. The *AOFV* and *ASFEs* values provided by MiSACO and MiSACO-Gower are provided in Table S-XIII of the supplementary file.

From Table S-XIII, MiSACO obtains better values than MiSACO-Gower on 22 (i.e., F1, F2, F4-F7, F10-F12, F14, F15, F17, F20, F21-F27, F29, and F30) and 21 (i.e., F1, F3-F6, F10, F11, F13-16, F19-F21, F23-F26, and F28-F30) test problems in terms of *AOFV* and in terms of *ASFEs*, respectively. According to the Wilcoxon's rank-sum test, MiSACO performs better than MiSACO-Gower on 12 test problems in terms of *AOFV* and 15 test problems in terms of *ASFEs*, respectively. Thus, it can be concluded that Gower distance may not be a good choice for MiSACO.

F. Influence of N_{min}

In MiSACO, N_{min} was used to decide whether the surrogate-assisted local search is implemented or not. The influence of N_{min} was investigated by experiments. In the investigation, six test problems (i.e., F1, F6, F13, F18, F24, and F29) were selected, and N_{min} was set to four different values: $1*n_1$, $5*n_1$, $10*n_1$, and $20*n_1$. The *AOFV* and *ASFEs* values are summarized in Table S-XIV. From Table S-XIV, when N_{min} is equal to $5*n_1$, MiSACO achieves the best performance.

S-III. Results

TABLE S-I
RESULTS OF ACO_{MV} AND MISACO OVER 20 INDEPENDENT RUNS ON THE CAPACITATED FACILITY LOCATION PROBLEMS. THE WILCOXON'S RANK-SUM TEST AT A 0.05 SIGNIFICANCE LEVEL WAS PERFORMED BETWEEN ACO_{MV} AND MISACO.

	ACO_{MV}		MiSACO
CFLP1	3192.08±2.30	+	**3185.42±2.21**
CFLP2	5803.43±16.93	+	**5776.52±5.39**
CFLP3	1404.17±30.73	+	**1381.19±2.93**
CFLP4	3442.87±163.90	+	**3378.06±127.74**
CFLP5	1080.75±33.51	+	**1050.35±12.54**
CFLP6	3356.49±646.10	+	**2550.43±117.73**
$+/-/\approx$	6/0/0		

TABLE S-II
RESULTS OF ACO_{MV} AND MISACO OVER 20 INDEPENDENT RUNS ON THE DUBINS TRAVELING SALESMAN PROBLEMS. THE WILCOXON'S RANK-SUM TEST AT A 0.05 SIGNIFICANCE LEVEL WAS PERFORMED BETWEEN ACO_{MV} AND MISACO.

	ACO_{MV}		MiSACO
DTSP1	492.97±31.51	+	**465.88±28.90**
DTSP2	696.08±60.62	+	**607.59±38.60**
DTSP3	914.11±25.17	+	**775.79±44.85**
DTSP4	635.81±25.87	+	**585.83±34.84**
DTSP5	960.07±36.03	+	**830.08±24.51**
DTSP6	1229.80±62.92	+	**1143.03±63.28**
$+/-/\approx$	6/0/0		

TABLE S-III
AOFV VALUES PROVIDED BY CAL-SAPSO, EGO-HAMMING, EGO-GOWER, BOA-RF, AND MISACO ON THE 30 ARTIFICIAL TEST PROBLEMS. THE WILCOXON'S RANK-SUM TEST AT A 0.05 SIGNIFICANCE LEVEL WAS PERFORMED BETWEEN MISACO AND EACH OF CAL-SAPSO, EGO-HAMMING, EGO-GOWER, AND BOA-RF.

	Problem	CAL-SAPSO		EGO-Hamming		EGO-Gower		BOA-RF		MiSACO
		AOFV ± *Std Dev*		*AOFV* ± *Std Dev*		*AOFV* ± *Std Dev*		*AOFV* ± *Std Dev*		*AOFV* ± *Std Dev*
Type 1	F1	8.68E+01 ± 9.85E+02	+	1.99E+01 ± 1.69E+01	+	4.69E+01 ± 6.07E+01	+	2.03E+03 ± 8.38E+02	+	**6.21E−08 ± 2.24E−08**
	F2	3.19E+01 ± 1.13E+01	+	5.35E+01 ± 1.38E+01	+	5.41E+01 ± 7.58E+00	+	8.24E+01 ± 8.04E+00	+	**2.59E+01 ± 1.19E+01**
	F3	1.62E+01 ± 1.17E−04	+	3.82E+00 ± 1.99E+00	+	3.85E+00 ± 4.63E+00	+	1.12E+01 ± 9.33E−01	+	**3.04E−01 ± 7.50E−01**
	F4	2.95E+00 ± 1.55E+01	+	9.94E−01 ± 6.50E−01	+	3.92E−01 ± 2.08E−01	+	4.13E+01 ± 1.45E+01	+	**4.51E−09 ± 2.25E−09**
	F5	1.54E+01 ± 1.10E+01	+	1.43E+00 ± 2.96E−01	+	1.10E+00 ± 1.98E−01	+	1.93E+01 ± 7.23E+00	+	**2.39E−01 ± 2.30E−01**
	F6	4.86E+02 ± 1.16E+03	+	2.80E+01 ± 1.75E+01	+	2.11E+02 ± 3.31E+02	+	2.31E+03 ± 6.37E+02	+	**5.74E−08 ± 2.50E−08**
	F7	3.23E+01 ± 1.34E+01	+	5.29E+01 ± 9.38E+00	+	4.84E+01 ± 7.35E+00	+	7.37E+01 ± 1.16E+01	+	**2.40E+01 ± 1.22E+01**
	F8	1.61E+01 ± 1.40E+00	+	3.64E+00 ± 3.40E+00	+	2.68E+00 ± 2.18E+00	+	1.21E+01 ± 1.18E+00	+	**7.27E−02 ± 2.83E−01**
	F9	6.48E+00 ± 1.92E+01	+	7.01E−01 ± 4.74E−01	+	2.34E+00 ± 3.69E+00	+	4.33E+01 ± 1.26E+01	+	**1.41E−06 ± 6.15E−06**
	F10	1.27E+01 ± 1.26E+01	+	1.35E+00 ± 2.38E−01	+	1.12E+00 ± 2.04E−01	+	2.73E+01 ± 5.34E+00	+	**2.75E−01 ± 3.01E−01**
Type 2	F11	3.68E+02 ± 5.08E+02	+	1.04E+01 ± 1.07E+01	+	8.29E−02 ± 1.26E−01	+	1.09E+02 ± 6.25E+01	+	**9.43E−08 ± 1.06E−07**
	F12	5.83E+01 ± 1.12E+01	+	4.60E+01 ± 1.74E+01	≈	**4.35E+01 ± 1.49E+01**	≈	6.49E+01 ± 6.79E+00	+	4.71E+01 ± 1.73E+01
	F13	8.54E+00 ± 1.21E+00	+	3.93E+00 ± 1.89E+00	+	5.71E−01 ± 2.31E−01	+	4.89E+00 ± 1.49E+00	+	**2.26E−04 ± 1.17E−04**
	F14	8.70E+00 ± 1.15E+01	+	3.34E−01 ± 5.37E−01	+	1.65E+00 ± 4.77E+00	+	8.53E−01 ± 4.34E−01	+	**2.04E−09 ± 2.03E−09**
	F15	8.92E+00 ± 8.34E+00	+	1.15E+00 ± 1.89E−01	+	3.49E−01 ± 1.63E−01	+	1.42E+00 ± 4.48E−01	+	**2.82E−07 ± 3.27E−07**
	F16	7.57E+02 ± 1.19E+03	+	1.19E+02 ± 1.72E+02	+	2.34E+01 ± 7.08E+01	+	1.07E+03 ± 2.45E+02	+	**1.27E+00 ± 5.66E+00**
	F17	6.98E+01 ± 1.37E+01	+	4.77E+01 ± 1.85E+01	≈	**4.11E+01 ± 7.67E+00**	≈	7.16E+01 ± 7.89E+00	+	4.50E+01 ± 1.61E+01
	F18	1.08E+01 ± 8.70E−01	+	3.30E+00 ± 2.52E+00	+	2.89E+00 ± 5.45E−01	≈	6.70E+00 ± 1.59E+00	+	**1.55E+00 ± 1.96E+00**
	F19	3.42E+01 ± 2.75E+01	+	7.46E+00 ± 1.03E+01	+	1.76E+00 ± 1.39E+00	+	6.88E+00 ± 3.21E+00	+	**2.71E−01 ± 7.16E−01**
	F20	1.40E+01 ± 1.98E+01	+	3.24E+00 ± 3.20E+00	+	3.01E+00 ± 2.74E+00	+	1.17E+01 ± 3.67E+00	+	**1.05E−01 ± 3.23E−01**
Type 3	F21	5.77E+02 ± 9.35E+02	+	7.82E+00 ± 4.26E+00	+	1.11E+01 ± 3.07E+01	+	1.01E+03 ± 5.16E+02	+	**7.60E−08 ± 5.64E−08**
	F22	5.37E+01 ± 1.41E+01	≈	4.51E+01 ± 8.61E+00	≈	**4.42E+01 ± 9.04E+00**	≈	7.34E+01 ± 1.37E+01	+	4.78E+01 ± 1.32E+01
	F23	1.26E+01 ± 1.19E+00	+	3.39E+00 ± 2.74E+00	+	3.97E+00 ± 5.04E+00	+	7.83E+00 ± 1.42E+00	+	**2.38E−04 ± 8.58E−05**
	F24	1.21E+01 ± 1.10E+01	+	1.87E−01 ± 1.53E−01	+	1.33E−01 ± 9.41E−02	+	1.79E+01 ± 5.85E+00	+	**1.18E−09 ± 4.54E−10**
	F25	2.26E+01 ± 9.22E+00	+	1.20E+00 ± 1.64E−01	+	9.86E−01 ± 4.11E−02	+	6.22E+00 ± 2.59E+00	+	**1.24E−01 ± 2.34E−01**
	F26	7.44E+02 ± 1.48E+03	+	1.20E+01 ± 7.74E+00	+	1.45E+02 ± 2.13E+02	+	1.73E+03 ± 5.08E+02	+	**9.70E−08 ± 8.28E−08**
	F27	5.64E+01 ± 1.13E+01	+	5.42E+01 ± 1.07E+01	+	5.07E+01 ± 8.96E+00	+	7.95E+01 ± 1.29E+01	+	**4.15E+01 ± 1.78E+01**
	F28	1.43E+01 ± 1.03E+00	+	2.65E+00 ± 7.17E−01	+	2.15E+00 ± 2.30E+00	+	9.70E+00 ± 1.12E+00	+	**9.76E−01 ± 1.46E+00**
	F29	1.52E+01 ± 2.40E+01	+	5.80E−01 ± 3.82E−01	+	1.80E+00 ± 1.59E+00	+	1.97E+01 ± 6.72E+00	+	**2.19E−05 ± 9.28E−05**
	F30	1.58E+01 ± 1.43E+01	+	1.19E+00 ± 1.17E−01	+	1.19E+00 ± 4.85E−01	+	1.64E+01 ± 5.16E+00	+	**4.92E−01 ± 6.18E−01**
+/−/≈		29/0/1		27/0/3		26/0/4		30/0/0		

TABLE S-IV

ASFEs VALUES PROVIDED BY CAL-SAPSO, EGO-HAMMING, EGO-GOWER, BOA-RF, AND MISACO ON THE 30 ARTIFICIAL TEST PROBLEMS. THE WILCOXON'S RANK-SUM TEST AT A 0.05 SIGNIFICANCE LEVEL WAS PERFORMED BETWEEN MISACO AND EACH OF CAL-SAPSO, EGO-HAMMING, EGO-GOWER, AND BOA-RF.

	Problem	CAL-SAPSO *ASFEs*±*Std Dev*		EGO-Hamming *ASFEs*±*Std Dev*		EGO-Gower *ASFEs*±*Std Dev*		BOA-RF *ASFEs*±*Std Dev*		MiSACO *ASFEs*±*Std Dev*
Type 1	F1	596.10±12.33	+	600.00±0.00	+	600.00±0.00	+	600.00±0.00	+	**249.95±36.49**
	F2	**600.00±0.00**	≈	**600.00±0.00**	≈	**600.00±0.00**	≈	**600.00±0.00**	+	**600.00±0.00**
	F3	600.00±0.00	+	600.00±0.00	+	600.00±0.00	+	600.00±0.00	+	**375.85±101.67**
	F4	562.20±55.38	+	599.50±0.53	+	585.80±9.38	+	600.00±0.00	+	**159.10±25.60**
	F5	600.00±0.00	+	600.00±0.00	+	599.80±0.63	+	600.00±0.00	+	**208.90±37.03**
	F6	600.00±0.00	+	600.00±0.00	+	600.00±0.00	+	600.00±0.00	+	**269.40±56.18**
	F7	**600.00±0.00**	≈	**600.00±0.00**	≈	**600.00±0.00**	≈	**600.00±0.00**	+	**600.00±0.00**
	F8	600.00±0.00	+	600.00±0.00	+	599.80±0.42	+	600.00±0.00	+	**391.60±63.22**
	F9	599.60±0.84	+	598.70±0.95	+	595.20±4.98	+	600.00±0.00	+	**231.15±65.29**
	F10	600.00±0.00	+	600.00±0.00	+	599.90±0.32	+	600.00±0.00	+	**270.20±123.87**
Type 2	F11	600.00±0.00	+	599.90±0.32	+	587.60±3.53	+	600.00±0.00	+	**285.75±73.21**
	F12	600.00±0.00	≈	**599.90±0.32**	≈	**599.50±1.58**	≈	**600.00±0.00**	+	600.00±0.00
	F13	600.00±0.00	+	599.90±0.32	+	594.10±4.20	+	559.80±37.78	+	**307.25±120.94**
	F14	600.00±0.00	+	596.30±2.21	+	527.50±49.70	+	584.90±47.75	+	**195.45±39.24**
	F15	599.90±0.32	+	599.90±0.32	+	587.60±3.34	+	600.00±0.00	+	**244.50±44.50**
	F16	600.00±0.00	+	600.00±0.00	+	598.80±1.23	+	600.00±0.00	+	**409.65±100.36**
	F17	**600.00±0.00**	≈	**600.00±0.00**	≈	**600.00±0.00**	≈	**600.00±0.00**	+	**600.00±0.00**
	F18	600.00±0.00	+	599.60±0.70	+	598.60±1.17	+	600.00±0.00	+	**538.25±93.69**
	F19	600.00±0.00	+	599.90±0.32	+	597.70±4.64	+	600.00±0.00	+	**356.45±122.27**
	F20	600.00±0.00	+	600.00±0.00	+	599.60±0.84	+	600.00±0.00	+	**401.70±109.00**
Type 3	F21	600.00±0.00	+	600.00±0.00	+	599.50±0.85	+	600.00±0.00	+	**279.55±36.60**
	F22	**600.00±0.00**	≈	**600.00±0.00**	≈	**600.00±0.00**	≈	**600.00±0.00**	+	**600.00±0.00**
	F23	600.00±0.00	+	600.00±0.00	+	599.40±1.07	+	600.00±0.00	+	**363.15±76.32**
	F24	600.00±0.00	+	595.30±1.34	+	526.50±58.93	+	600.00±0.00	+	**185.15±37.79**
	F25	600.00±0.00	+	600.00±0.00	+	599.50±0.53	+	600.00±0.00	+	**229.45±32.75**
	F26	600.00±0.00	+	600.00±0.00	+	599.90±0.32	+	600.00±0.00	+	**330.80±62.07**
	F27	**600.00±0.00**	≈	**600.00±0.00**	≈	**600.00±0.00**	≈	**600.00±0.00**	+	**600.00±0.00**
	F28	600.00±0.00	+	600.00±0.00	+	599.50±0.53	+	600.00±0.00	+	**475.15±117.83**
	F29	599.70±0.95	+	597.40±1.90	+	596.30±6.04	+	600.00±0.00	+	**273.10±83.76**
	F30	600.00±0.00	+	600.00±0.00	+	599.50±0.53	+	600.00±0.00	+	**353.50±118.11**
$+/-/\approx$		24/0/6		24/0/6		24/0/6		24/0/6		

TABLE S-V
AOFV VALUES PROVIDED BY CAL-SAPSO, EGO-HAMMING, EGO-GOWER, BOA-RF, AND MISACO ON THE SIX CAPACITATED FACILITY LOCATION PROBLEMS. THE WILCOXON'S RANK-SUM TEST AT A 0.05 SIGNIFICANCE LEVEL WAS PERFORMED BETWEEN MISACO AND EACH OF CAL-SAPSO, EGO-HAMMING, EGO-GOWER, AND BOA-RF.

Problem	CAL-SAPSO		EGO-Hamming		EGO-Gower		BOA-RF		MiSACO
	AOFV±*Std Dev*		*AOFV*±*Std Dev*		*AOFV*±*Std Dev*		*AOFV*±*Std Dev*		*AOFV*±*Std Dev*
CFLP1	3203.22±2.20	+	3195.73±2.51	+	3191.68±1.76	+	3192.56±1.64	+	**3185.42±2.21**
CFLP2	5808.89±11.56	+	5800.91±10.08	+	5797.36±12.44	+	5795.30±11.89	+	**5776.52±5.39**
CFLP3	1403.34±9.37	+	1393.46±7.28	+	1387.62±2.26	+	1388.76±2.97	+	**1381.19±2.93**
CFLP4	3620.66±133.98	+	3659.44±115.64	+	**3356.00±125.48**	≈	3500.90±117.52	≈	3378.06±127.74
CFLP5	1090.61±98.20	+	1110.07±70.00	+	1053.74±2.75	≈	1050.95±3.37	≈	**1050.35±12.54**
CFLP6	3124.20±277.01	+	3098.86±222.44	+	2553.93±37.60	≈	3279.56±447.19	+	**2550.43±117.73**
$+/-/\approx$	6/0/0		6/0/0		3/0/3		5/0/1		

TABLE S-VI
AOFV VALUES PROVIDED BY CAL-SAPSO, EGO-HAMMING, EGO-GOWER, BOA-RF, AND MISACO ON THE SIX DUBINS TRAVELING SALESMAN PROBLEMS. THE WILCOXON'S RANK-SUM TEST AT A 0.05 SIGNIFICANCE LEVEL WAS PERFORMED BETWEEN MISACO AND EACH OF CAL-SAPSO, EGO-HAMMING, EGO-GOWER, AND BOA-RF.

Problem	CAL-SAPSO		EGO-Hamming		EGO-Gower		BOA-RF		MiSACO
	AOFV±*Std Dev*		*AOFV*±*Std Dev*		*AOFV*±*Std Dev*		*AOFV*±*Std Dev*		*AOFV*±*Std Dev*
DTSP1	490.55±10.23	+	499.08±25.95	+	479.46±17.36	+	495.81±9.81	+	**465.88±28.90**
DTSP2	720.20±22.30	+	739.70±35.04	+	705.55±24.88	+	718.00±19.80	+	**607.59±38.60**
DTSP3	923.73±23.21	+	952.06±28.13	+	932.76±31.01	+	975.70±11.24	+	**775.79±44.85**
DTSP4	667.26±14.77	+	674.05±34.42	+	629.15±16.22	+	644.97±3.45	+	**585.83±34.84**
DTSP5	1001.56±35.76	+	1011.25±33.53	+	906.68±34.32	+	1011.17±36.56	+	**830.08±24.51**
DTSP6	1377.33±36.28	+	1322.72±26.60	+	1327.63±52.65	+	1336.04±32.67	+	**1143.03±63.28**
$+/-/\approx$	6/0/0		6/0/0		6/0/0		6/0/0		

TABLE S-VII

AOFV VALUES PROVIDED BY MISACO-LSBT-RAND, MISACO-RBF-RAND, MISACO-LSBT-RBF, AND MISACO ON THE 30 ARTIFICIAL TEST PROBLEMS. THE WILCOXON'S RANK-SUM TEST AT A 0.05 SIGNIFICANCE LEVEL WAS PERFORMED BETWEEN MISACO AND EACH OF MISACO-LSBT-RAND, MISACO-RBF-RAND, AND MISACO-LSBT-RBF.

	Problem	MiSACO-LSBT-Rand *AOFV* ± *Std Dev*		MiSACO-RBF-Rand *AOFV* ± *Std Dev*		MiSACO-LSBT-RBF *AOFV* ± *Std Dev*		MiSACO *AOFV* ± *Std Dev*
Type 1	F1	6.31E−08 ± 3.55E−08	≈	**4.59E−08** ± **2.82E−08**	≈	5.30E−08 ± 1.68E−08	≈	6.21E−08 ± 2.24E−08
	F2	2.83E+01 ± 9.93E+00	≈	2.91E+01 ± 1.71E+01	≈	3.29E+01 ± 9.83E+00	≈	**2.59E+01** ± **1.19E+01**
	F3	4.53E−01 ± 1.12E+00	+	**2.38E−01** ± **7.61E−01**	≈	7.13E−01 ± 1.19E+00	≈	3.04E−01 ± 7.50E−01
	F4	7.10E−06 ± 3.17E−05	+	1.36E−08 ± 4.64E−08	≈	**2.49E−09** ± **1.68E−09**	≈	4.51E−09 ± 2.25E−09
	F5	3.63E−01 ± 2.82E−01	≈	**2.24E−01** ± **2.51E−01**	≈	2.49E−01 ± 2.52E−01	≈	2.39E−01 ± 2.30E−01
	F6	9.72E−08 ± 8.21E−08	+	3.95E+00 ± 1.64E+01	+	7.49E−08 ± 2.79E−08	≈	**5.74E−08** ± **2.50E−08**
	F7	2.82E+01 ± 1.13E+01	≈	2.84E+01± 1.33E+01	≈	2.77E+01 ± 1.17E+01	≈	**2.40E+01** ± **1.22E+01**
	F8	8.68E−01 ± 1.36E+00	+	1.68E−01 ± 5.09E−01	≈	3.21E−01 ± 8.36E−01	+	**7.27E−02** ± **2.83E−01**
	F9	2.85E−02 ± 8.02E−02	+	1.69E−01 ± 6.07E−01	+	1.34E−01 ± 6.00E−01	+	**1.41E−06** ± **6.15E−06**
	F10	2.99E−01 ± 3.33E−01	≈	3.00E−01 ± 3.20E−01	≈	5.84E−01 ± 1.68E+00	≈	**2.75E−01** ± **3.01E−01**
Type 2	F11	6.10E−01 ± 1.73E+00	≈	2.77E+00 ± 1.24E+01	+	6.95E+01 ± 1.64E+02	+	**9.43E−08** ± **1.06E−07**
	F12	4.82E+01 ± 1.12E+01	≈	5.48E+01 ± 2.11E+01	+	4.85E+01 ± 6.29E+00	≈	**4.71E+01** ± **1.73E+01**
	F13	**1.85E−04** ± **7.55E−05**	≈	1.14E+00 ± 1.88E+00	≈	2.28E−01 ± 1.01E+00	≈	2.26E−04 ± 1.17E−04
	F14	2.83E−03 ± 7.68E−03	≈	2.15E−01 ± 4.17E−01	+	**1.36E−09** ± **1.97E−09**	≈	2.04E−09 ± 2.03E−09
	F15	7.94E−02 ± 2.43E−01	+	2.81E−01 ± 7.08E−01	≈	5.73E−07 ± 6.90E−07	≈	**2.82E−07** ± **3.27E−07**
	F16	2.36E+01 ± 8.55E+01	≈	1.46E+02 ± 2.24E+02	+	1.84E+02 ± 5.86E+02	+	**1.27E+00** ± **5.66E+00**
	F17	5.35E+01 ± 1.23E+01	≈	5.15E+01 ± 1.40E+01	≈	5.01E+01 ± 9.29E+00	≈	**4.50E+01** ± **1.61E+01**
	F18	**8.43E−01** ± **1.42E+00**	≈	3.38E+00 ± 2.15E+00	+	3.17E+00 ± 3.48E+00	≈	1.55E+00 ± 1.96E+00
	F19	**1.86E−01** ± **3.10E−01**	≈	1.53E+00 ± 1.58E+00	+	1.36E+00 ± 3.36E+00	+	2.71E−01 ± 7.16E−01
	F20	3.27E−01 ± 4.03E−01	+	3.24E+00 ± 3.61E+00	+	3.81E+00 ± 1.22E+01	+	**1.05E−01** ± **3.23E−01**
Type 3	F21	2.86E−01 ± 1.26E+00	≈	3.95E+00 ± 1.64E+01	+	2.24E+01 ± 1.00E+02	+	**7.60E−08** ± **5.64E−08**
	F22	4.81E+01 ± 1.27E+01	≈	5.08E+01 ± 1.67E+01	≈	5.11E+01 ± 1.62E+01	≈	**4.78E+01** ± **1.32E+01**
	F23	1.65E−01 ± 6.52E−02	+	1.86E−01 ± 8.31E−01	+	5.23E−02 ± 2.33E−01	+	**2.38E−04** ± **8.58E−05**
	F24	5.01E−04 ± 2.24E−03	+	1.04E−01 ± 3.21E−01	+	1.39E−09 ± 7.64E−10	≈	**1.18E−09** ± **4.54E−10**
	F25	3.19E−01 ± 3.33E−01	+	3.43E−01 ± 3.77E−01	≈	1.51E−01 ± 2.75E−01	+	**1.24E−01** ± **2.34E−01**
	F26	9.85E−01 ± 4.40E+00	≈	2.72E+01 ± 7.85E+01	+	6.27E+01 ± 1.57E+02	+	**9.70E−08** ± **8.28E−08**
	F27	4.34E+01 ± 1.63E+01	≈	4.39E+01 ± 1.15E+01	≈	4.30E+01 ± 1.48E+01	≈	**4.15E+01** ± **1.78E+01**
	F28	1.96E+00 ± 1.93E+00	≈	1.49E+00 ± 1.67E+00	≈	1.54E+00 ± 1.56E+00	≈	**9.76E−01** ± **1.46E+00**
	F29	1.20E−01 ± 2.36E−01	+	4.98E−01 ± 9.01E−01	+	9.58E−01 ± 2.96E+00	≈	**2.19E−05**± **9.28E−05**
	F30	6.69E−01 ± 3.49E−01	+	8.20E−01 ± 7.84E−01	≈	5.30E−01 ± 3.74E−01	≈	**4.92E−01** ± **6.18E−01**
+/−/≈		12/0/18		14/0/16		10/0/20		

TABLE S-VIII
ASFEs VALUES PROVIDED BY MISACO-LSBT-RAND, MISACO-RBF-RAND, MISACO-LSBT-RBF, AND MISACO ON THE 30 ARTIFICIAL TEST PROBLEMS. THE WILCOXON'S RANK-SUM TEST AT A 0.05 SIGNIFICANCE LEVEL WAS PERFORMED BETWEEN MISACO AND EACH OF MISACO-LSBT-RAND, MISACO-RBF-RAND, AND MISACO-LSBT-RBF.

	Problem	MiSACO-LSBT-Rand		MiSACO-RBF-Rand		MiSACO-LSBT-RBF		MiSACO
		ASFEs ± *Std Dev*		*ASFEs* ± *Std Dev*		*ASFEs* ± *Std Dev*		*ASFEs* ± *Std Dev*
Type 1	F1	289.45 ± 51.18	+	**217.30 ± 75.74**	≈	221.60 ± 30.33	≈	249.95 ± 36.49
	F2	**600.00 ± 0.00**	≈	**600.00 ± 0.00**	≈	600.00 ± 0.00	≈	**600.00 ± 0.00**
	F3	362.85 ± 107.46	≈	**350.25 ± 97.75**	≈	400.70 ± 136.77	+	355.85 ± 101.67
	F4	223.00 ± 65.80	+	168.50 ± 65.50	≈	**150.25 ± 24.40**	≈	159.10 ± 25.60
	F5	291.85 ± 84.61	+	227.45 ± 60.24	≈	215.40 ± 47.74	≈	**208.90 ± 37.03**
	F6	368.25 ± 75.77	+	303.00 ± 139.55	≈	273.65 ± 84.48	≈	**269.40 ± 56.18**
	F7	**600.00 ± 0.00**	≈	**600.00 ± 0.00**	≈	**600.00 ± 0.00**	≈	**600.00 ± 0.00**
	F8	419.55 ± 133.02	≈	380.65 ± 89.38	≈	**368.35 ± 117.16**	≈	391.60 ± 63.22
	F9	260.55 ± 54.94	+	236.80 ± 146.13	≈	**212.25 ± 103.96**	≈	231.15 ± 65.29
	F10	342.05 ± 77.51	+	327.55 ± 128.06	≈	282.90 ± 114.52	≈	**270.20 ± 123.87**
Type 2	F11	467.35 ± 77.98	+	371.20 ± 121.88	+	352.80 ± 159.45	+	**285.75 ± 73.21**
	F12	**600.00 ± 0.00**	≈	**600.00 ± 0.00**	≈	**600.00 ± 0.00**	≈	**600.00 ± 0.00**
	F13	**293.45 ± 84.69**	≈	438.20 ± 128.56	+	307.95 ± 113.14	≈	307.25 ± 120.94
	F14	304.75 ± 53.62	+	314.30 ± 141.19	+	201.00 ± 71.51	≈	**195.45 ± 39.24**
	F15	413.40 ± 56.67	+	380.00 ± 146.59	+	246.50 ± 57.68	≈	**244.50 ± 44.50**
	F16	510.05 ± 78.44	+	538.90 ± 96.12	+	464.55 ± 147.50	+	**409.65 ± 100.36**
	F17	**600.00 ± 0.00**	≈	**600.00 ± 0.00**	≈	**600.00 ± 0.00**	≈	**600.00 ± 0.00**
	F18	**432.75 ± 137.72**	≈	586.90 ± 29.26	≈	522.60 ± 131.29	≈	538.25 ± 93.69
	F19	399.35 ± 65.53	+	509.40 ± 135.33	+	366.25 ± 156.84	≈	**356.45 ± 122.27**
	F20	484.45 ± 69.50	≈	556.15 ± 83.22	+	410.15 ± 142.85	≈	**401.70 ± 109.14**
Type 3	F21	439.95 ± 79.69	+	350.25 ± 127.82	≈	290.25 ± 121.50	≈	**279.55 ± 36.60**
	F22	**600.00 ± 0.00**	≈	**600.00 ± 0.00**	≈	600.00 ± 0.00	≈	**600.00 ± 0.00**
	F23	380.35 ± 42.08	+	373.00 ± 73.12	≈	379.65 ± 82.56	≈	**363.15 ± 76.32**
	F24	328.05 ± 54.58	+	265.80 ± 132.05	+	194.65 ± 46.44	≈	**185.15 ± 37.79**
	F25	373.80 ± 54.57	+	309.60 ± 75.25	+	**222.90 ± 74.75**	≈	229.45 ± 32.75
	F26	458.85 ± 80.85	+	452.85 ± 113.08	+	351.35 ± 153.04	≈	**330.80 ± 62.07**
	F27	**600.00 ± 0.00**	≈	**600.00 ± 0.00**	≈	**600.00 ± 0.00**	≈	**600.00 ± 0.00**
	F28	518.25 ± 106.08	≈	520.15 ± 98.98	≈	518.60 ± 118.47	+	**475.15 ± 117.83**
	F29	405.20 ± 66.87	+	361.35 ± 148.74	≈	304.05 ± 153.11	≈	**273.10 ± 83.76**
	F30	479.40 ± 79.15	+	453.95 ± 121.65	+	375.05 ± 148.82	+	**353.50 ± 118.11**
$+/-/\approx$		18/0/12		11/0/19		5/0/25		

TABLE S-IX
RESULTS PROVIDED BY MISACO-WOLOCAL AND MISACO ON THE 30 ARTIFICIAL TEST PROBLEMS. THE WILCOXON'S RANK-SUM TEST AT A 0.05 SIGNIFICANCE LEVEL WAS PERFORMED BETWEEN MISACO AND MISACO-WOLOCAL.

	Problem	MiSACO-WoLocal *AOFV ± Std Dev*		MiSACO *AOFV ± Std Dev*	MiSACO-WoLocal *ASFEs ± Std Dev*		MiSACO *ASFEs ± Std Dev*
Type 1	F1	1.84E−02 ± 1.18E−02	+	**6.21E−08 ± 2.24E−08**	319.40 ± 21.71	+	**249.95 ± 36.49**
	F2	5.57E+01 ± 8.86E+00	+	**2.59E+01 ± 1.19E+01**	**600.00 ± 0.00**	≈	**600.00 ± 0.00**
	F3	8.09E−01 ± 4.02E−01	+	**3.04E−01 ± 7.50E−01**	563.50 ± 41.66	+	**355.85 ± 101.67**
	F4	2.98E−04 ± 3.12E−04	+	**4.51E−09 ± 2.25E−09**	177.35 ± 23.13	≈	**159.10 ± 25.60**
	F5	3.57E−01 ± 1.40E−01	≈	**2.39E−01 ± 2.30E−01**	286.15 ± 26.90	+	**208.90 ± 37.03**
	F6	1.38E−02 ± 9.91E−03	+	**5.74E−08 ± 2.50E−08**	320.45 ± 21.41	+	**269.40 ± 56.18**
	F7	5.47E+01 ± 5.41E+00	+	**2.40E+01 ± 1.22E+01**	**600.00 ± 0.00**	≈	**600.00 ± 0.00**
	F8	9.64E−01 ± 3.63E−01	+	**7.27E−02 ± 2.83E−01**	576.15 ± 28.62	+	**391.60 ± 63.22**
	F9	6.81E−04 ± 5.08E−04	+	**1.41E−06 ± 6.15E−06**	247.20 ± 29.33	≈	**231.15 ± 65.29**
	F10	3.00E−01 ± 1.79E−01	≈	**2.75E−01 ± 3.01E−01**	305.30 ± 40.98	≈	**270.20 ± 123.87**
Type 2	F11	2.93E−04 ± 3.80E−04	+	**9.43E−08 ± 1.06E−07**	287.50 ± 75.80	≈	**285.75 ± 73.21**
	F12	4.82E+01 ± 1.99E+01	≈	**4.71E+01 ± 1.73E+01**	**600.00 ± 0.00**	≈	**600.00 ± 0.00**
	F13	1.23E−02 ± 9.72E−03	+	**2.26E−04 ± 1.17E−04**	316.55 ± 63.82	≈	**307.25 ± 120.94**
	F14	3.72E−02 ± 1.66E−01	+	**2.04E−09 ± 2.03E−09**	226.15 ± 58.14	≈	**195.45 ± 39.24**
	F15	2.39E−02 ± 1.95E−02	+	**2.82E−07 ± 3.27E−07**	261.10 ± 50.52	≈	**244.50 ± 44.50**
	F16	5.92E+00 ± 1.88E+01	+	**1.27E+00 ± 5.66E+00**	**396.30 ± 129.11**	≈	409.65 ± 100.36
	F17	5.02E+01 ± 1.73E+01	≈	**4.50E+01 ± 1.61E+01**	**600.00 ± 0.00**	≈	**600.00 ± 0.00**
	F18	2.30E+00 ± 1.91E+00	≈	**1.55E+00 ± 1.96E+00**	554.20 ± 93.19	+	**538.25 ± 93.69**
	F19	2.93E−01 ± 5.50E−01	+	**2.71E−01 ± 7.16E−01**	366.65 ± 135.86	≈	**356.45 ± 122.27**
	F20	1.95E−01 ± 4.06E−01	+	**1.05E−01 ± 3.23E−01**	405.70 ± 109.40	≈	**401.70 ± 109.14**
Type 3	F21	2.53E−03 ± 2.49E−03	+	**7.60E−08 ± 5.64E−08**	294.40 ± 40.80	≈	**279.55** ± 36.60
	F22	4.99E+01 ± 1.20E+01	+	**4.78E+01 ± 1.32E+01**	**600.00 ± 0.00**	≈	**600.00 ± 0.00**
	F23	2.77E−01 ± 1.63E−01	≈	**2.38E−04 ± 8.58E−05**	459.10 ± 49.46	+	**363.15 ± 76.32**
	F24	1.78E−04 ± 1.46E−04	+	**1.18E−09 ± 4.54E−10**	199.80 ± 35.88	≈	**185.15 ± 37.79**
	F25	**6.21E−02 ± 5.41E−02**	≈	1.24E−01 ± 2.34E−01	254.00 ± 22.53	+	**229.45 ± 32.75**
	F26	2.35E−03 ± 2.34E−03	+	**9.70E−08 ± 8.28E−08**	333.25 ± 64.27	≈	**330.80** ± 62.07
	F27	4.81E+01 ± 1.46E+01	≈	**4.15E+01 ± 1.78E+01**	**600.00 ± 0.00**	≈	**600.00 ± 0.00**
	F28	9.89E−01 ± 9.57E−01	≈	**9.76E−01 ± 1.46E+00**	528.65 ± 63.43	+	**475.15 ± 117.83**
	F29	1.12E−03 ± 1.09E−03	+	**2.19E−05 ± 9.28E−05**	278.40 ± 56.19	≈	**273.10 ± 83.76**
	F30	**1.62E−01 ± 1.49E−01**	≈	4.92E−01 ± 6.18E−01	369.75 ± 80.04	≈	**353.50 ± 118.11**
+/−/≈		20/0/10			9/0/21		

TABLE S-X
RESULTS PROVIDED BY MISACO-KRIGING AND MISACO ON THE 30 ARTIFICIAL TEST PROBLEMS. THE WILCOXON'S RANK-SUM TEST AT A 0.05 SIGNIFICANCE LEVEL WAS PERFORMED BETWEEN MISACO-KRIGING AND MISACO.

	Problem	MiSACO-Kriging *AOFV ± Std Dev*		MiSACO *AOFV ± Std Dev*	MiSACO-Kriging *ASFEs ± Std Dev*		MiSACO *ASFEs ± Std Dev*
Type 1	F1	6.72E-08±2.21E-08	≈	**6.21E-08±2.24E-08**	288.40 ±58.96	≈	**249.95 ±36.49**
	F2	**2.44E+01±7.32E+00**	≈	2.59E+01±1.19E+01	**600.00 ±0.00**	≈	**600.00 ±0.00**
	F3	4.38E-01±3.09E-01	≈	**3.04E-01±7.50E-01**	**313.40 ±145.04**	−	375.85 ±101.67
	F4	1.53E-08±1.56E-08	≈	**4.51E-09±2.25E-09**	197.40 ±32.22	≈	**159.10 ±25.60**
	F5	2.62E-01±9.93E-02	≈	**2.39E-01±2.30E-01**	274.60 ±70.70	+	**208.90 ±37.03**
	F6	7.21E-08±1.27E-08	≈	**5.74E-08±2.50E-08**	305.60 ±51.49	≈	**269.40 ±56.18**
	F7	**2.35E+01±4.97E+00**	≈	2.40E+01±1.22E+01	**600.00 ±0.00**	≈	**600.00 ±0.00**
	F8	9.54E-01±2.00E+00	≈	**7.27E-02±2.83E-01**	418.40 ±152.48	≈	**391.60 ±63.22**
	F9	**1.85E-03±2.22E-03**	≈	2.59E-03±8.51E-03	**201.20 ±53.13**	≈	231.15 ±65.29
	F10	**1.67E-01±1.24E-01**	≈	2.75E-01±3.01E-01	**262.20 ±53.45**	≈	270.20 ±123.87
Type 2	F11	**8.68E-08±5.66E-08**	≈	9.43E-08±1.06E-07	315.60 ±46.72	≈	**285.75 ±73.21**
	F12	4.86E+01±1.19E+01	≈	**4.71E+01±1.73E+01**	**600.00 ±0.00**	≈	**600.00 ±0.00**
	F13	3.22E-01±2.83E-05	≈	**3.19E-01±1.29E+00**	**257.20 ±76.04**	−	307.25 ±120.94
	F14	**1.10E-09±9.15E-10**	≈	2.04E-09±2.03E-09	229.00 ±75.72	≈	**195.45 ±39.24**
	F15	1.26E-01±2.82E-01	≈	**2.82E-07±3.27E-07**	279.00 ±45.29	≈	**244.50 ±44.50**
	F16	**9.86E-01±8.69E-08**	≈	1.27E+00±5.66E+00	411.40 ±79.02	≈	**409.65 ±100.36**
	F17	6.01E+01±6.82E+00	≈	**4.50E+01±1.61E+01**	**600.00 ±0.00**	≈	**600.00 ±0.00**
	F18	**1.41E+00±2.00E+00**	≈	1.55E+00±1.96E+00	**503.20 ±171.32**	≈	538.25 ±93.69
	F19	3.53E-01±6.79E-08	≈	**2.71E-01±7.16E-01**	399.00 ±80.64	≈	**356.45 ±122.27**
	F20	1.86E-01±4.05E-01	≈	**1.05E-01±3.23E-01**	**361.60 ±96.96**	≈	401.70 ±109.00
Type 3	F21	8.13E-08±5.73E-08	≈	**7.60E-08±5.64E-08**	346.00 ±39.24	+	**279.55 ±36.60**
	F22	**3.42E+01±6.33E+00**	≈	4.78E+01±1.32E+01	**600.00 ±0.00**	≈	**600.00 ±0.00**
	F23	5.31E-01±1.19E+00	≈	**1.52E-01±6.77E-01**	367.60 ±152.87	≈	**363.15 ±76.32**
	F24	**1.33E-09±1.07E-09**	≈	9.90E-07±4.42E-06	276.40 ±28.97	+	**185.15 ±37.79**
	F25	1.90E-01±2.75E-01	≈	**1.24E-01±2.34E-01**	244.40 ±40.46	≈	**229.45 ±32.75**
	F26	**6.63E-08±3.52E-08**	≈	9.70E-08±8.28E-08	391.00 ±50.31	≈	**330.80 ±62.07**
	F27	4.33E+01±1.05E+01	≈	**4.15E+01±1.78E+01**	**600.00 ±0.00**	≈	**600.00 ±0.00**
	F28	**1.75E-02±3.84E-02**	≈	9.76E-01±1.46E+00	**345.00 ±141.24**	−	475.15 ±117.83
	F29	**1.60E-03±3.58E-03**	≈	3.74E-02±1.44E-01	322.20 ±71.29	≈	**273.10 ±83.76**
	F30	5.84E-01±8.77E-02	≈	**4.92E-01±6.18E-01**	354.20 ±62.60	≈	**353.50 ±118.11**
$+/-/\approx$		0/0/30			3/3/24		

TABLE S-XI
RUNTIME OF MISACO-KRIGING AND MISACO ON THE 30 ARTIFICIAL TEST PROBLEMS.

	Type 1			Type 2			Type 3	
	Runtime (second)							
	MiSACO-Kriging	MiSACO		MiSACO-Kriging	MiSACO		MiSACO-Kriging	MiSACO
F1	200.78	102.18	F11	357.74	111.55	F21	211.83	107.96
F2	183.81	99.67	F12	338.41	111.41	F22	193.26	113.60
F3	178.98	105.40	F13	343.88	112.21	F23	216.68	102.01
F4	189.00	101.29	F14	374.83	111.74	F24	329.88	113.24
F5	190.74	99.18	F15	385.36	112.60	F25	335.04	109.60
F6	199.48	99.86	F16	227.33	118.48	F26	204.69	116.34
F7	179.61	97.36	F17	217.48	118.14	F27	189.81	118.49
F8	181.95	101.80	F18	211.56	117.07	F28	208.51	110.04
F9	188.57	101.99	F19	224.43	119.36	F29	201.70	118.56
F10	189.65	102.20	F20	220.73	116.61	F30	200.65	112.46

TABLE S-XII
RESULTS PROVIDED BY MISACO-RF AND MISACO ON THE 30 ARTIFICIAL TEST PROBLEMS. THE WILCOXON'S RANK-SUM TEST AT A 0.05 SIGNIFICANCE LEVEL WAS PERFORMED BETWEEN MISACO-RF AND MISACO.

	Problem	MiSACO-RF *AOFV ± Std Dev*		MiSACO *AOFV ± Std Dev*	MiSACO-RF *ASFEs ± Std Dev*		MiSACO *ASFEs ± Std Dev*
Type 1	F1	3.26E-06±1.34E-05	+	**6.21E-08±2.24E-08**	342.35 ±61.50	+	**249.95 ±36.49**
	F2	2.64E+01±9.70E+00	≈	**2.59E+01±1.19E+01**	**600.00 ±0.00**	≈	**600.00 ±0.00**
	F3	4.02E-01±8.71E-01	+	**3.04E-01±7.50E-01**	403.80 ±105.87	≈	**375.85 ±101.67**
	F4	4.37E-05±1.30E-04	+	**4.51E-09±2.25E-09**	241.45 ±80.73	+	**159.10 ±25.60**
	F5	**2.28E-01±2.18E-01**	≈	2.39E-01±2.30E-01	309.45 ±83.45	+	**208.90 ±37.03**
	F6	7.03E-08±7.51E+01	+	**5.74E-08±2.50E-08**	310.95 ±53.85	+	**269.40 ±56.18**
	F7	2.55E+01±1.41E+01	≈	**2.40E+01±1.22E+01**	**600.00 ±0.00**	≈	**600.00 ±0.00**
	F8	1.06E+00±1.92E+00	+	**7.27E-02±2.83E-01**	468.05 ±137.25	+	**391.60 ±63.22**
	F9	6.17E-02±8.55E-01	+	**2.59E-03±8.51E-03**	251.05 ±107.95	≈	**231.15 ±65.29**
	F10	3.18E-01±6.15E-01	+	**2.75E-01±3.01E-01**	320.55 ±84.50	+	**270.20 ±123.87**
Type 2	F11	3.74E+01±1.48E+02	+	**9.43E-08±1.06E-07**	454.20 ±139.78	+	**285.75 ±73.21**
	F12	4.82E+01±1.56E+01	≈	**4.71E+01±1.73E+01**	**600.00 ±0.00**	≈	**600.00 ±0.00**
	F13	9.25E-01±1.47E+00	+	**3.19E-01±1.29E+00**	445.50 ±145.48	+	**307.25 ±120.94**
	F14	2.24E-01±6.44E-01	+	**2.04E-09±2.03E-09**	297.25 ±150.56	+	**195.45 ±39.24**
	F15	6.73E-01±1.12E+00	+	**2.82E-07±3.27E-07**	443.35 ±143.74	+	**244.50 ±44.50**
	F16	9.92E+01±3.05E-08	+	**1.27E+00±5.66E+00**	548.70 ±71.57	+	**409.65 ±100.36**
	F17	5.48E+01±1.07E+01	≈	**4.50E+01±1.61E+01**	**600.00 ±0.00**	≈	**600.00 ±0.00**
	F18	1.99E+00±1.21E+00	+	**1.55E+00±1.96E+00**	546.60 ±83.82	≈	**538.25 ±93.69**
	F19	7.98E-01±2.63E-01	+	**2.71E-01±7.16E-01**	476.55 ±135.21	+	**356.45 ±122.27**
	F20	1.69E+00±2.36E-01	+	**1.05E-01±3.23E-01**	542.80 ±83.76	+	**401.70 ±109.00**
Type 3	F21	4.08E+00±1.53E+01	+	**7.60E-08±5.64E-08**	493.20 ±92.32	+	**279.55 ±36.60**
	F22	**3.92E+01±1.06E+01**	≈	4.78E+01±1.32E+01	**600.00 ±0.00**	≈	**600.00 ±0.00**
	F23	1.06E+00±1.49E+00	+	**1.52E-01±6.77E-01**	451.60 ±118.53	+	**363.15 ±76.32**
	F24	1.97E-01±3.74E-01	+	**9.90E-07±4.42E-06**	363.00 ±118.42	+	**185.15 ±37.79**
	F25	5.62E-01±3.73E-01	+	**1.24E-01±2.34E-01**	376.75 ±121.34	+	**229.45 ±32.75**
	F26	2.23E+01±1.56E+02	+	**9.70E-08±8.28E-08**	432.85 ±136.29	+	**330.80 ±62.07**
	F27	**3.40E+01±1.25E+01**	≈	4.15E+01±1.78E+01	**600.00 ±0.00**	≈	**600.00 ±0.00**
	F28	1.68E+00±2.02E+00	+	**9.76E-01±1.46E+00**	475.85 ±125.74	≈	**475.15 ±117.83**
	F29	5.49E-01±9.06E-01	+	**3.74E-02±1.44E-01**	428.45 ±134.31	+	**273.10 ±83.76**
	F30	7.94E-01±2.57E+00	+	**4.92E-01±6.18E-01**	459.15 ±109.33	+	**353.50 ±118.11**
$+/-/\approx$		23/0/7			20/0/10		

TABLE S-XIII
RESULTS PROVIDED BY MISACO-GOWER AND MISACO ON THE 30 ARTIFICIAL TEST PROBLEMS. THE WILCOXON'S RANK-SUM TEST AT A 0.05 SIGNIFICANCE LEVEL WAS PERFORMED BETWEEN MISACO-GOWER AND MISACO.

	Problem	MiSACO-Gower *AOFV ± Std Dev*		MiSACO *AOFV ± Std Dev*	MiSACO-Gower *ASFEs ± Std Dev*		MiSACO *ASFEs ± Std Dev*
Type 1	F1	1.16E-04±2.99E-04	+	**6.21E-08±2.24E-08**	382.90±110.05	+	**249.95±36.49**
	F2	3.16E+01±7.91E+00	≈	**2.59E+01±1.19E+01**	**600.00±0.00**	≈	**600.00±0.00**
	F3	**2.35E-01±4.92E-01**	≈	3.04E-01±7.50E-01	448.40±92.61	≈	**355.85±101.67**
	F4	5.97E-07±1.25E-06	+	**4.51E-09±2.25E-09**	233.70±46.96	+	**159.10±25.60**
	F5	3.40E-01±2.08E-01	≈	**2.39E-01±2.30E-01**	319.10±57.36	+	**208.90±37.03**
	F6	2.98E-07±6.00E-07	+	**5.74E-08±2.50E-08**	340.20±69.41	≈	**269.40±56.18**
	F7	2.42E+01±6.68E+00	≈	**2.40E+01±1.22E+01**	**600.00±0.00**	≈	**600.00±0.00**
	F8	**3.55E-03±3.97E-03**	−	7.27E-02±2.83E-01	**378.70±38.99**	≈	391.60±63.22
	F9	**2.00E-06±2.88E-06**	−	2.59E-03±8.51E-03	**227.20±38.02**	≈	231.15±65.29
	F10	4.22E-01±2.17E-01	≈	**2.75E-01±3.01E-01**	289.50±79.86	≈	**270.20±123.87**
Type 2	F11	1.19E+00±1.54E+00	+	**9.43E-08±1.06E-07**	504.30±107.42	+	**285.75±73.21**
	F12	5.03E+01±1.04E+01	≈	**4.71E+01±1.73E+01**	**600.00±0.00**	≈	**600.00±0.00**
	F13	**4.32E-02±1.36E-01**	−	4.19E-01±1.29E+00	316.10±64.65	≈	**307.25±120.94**
	F14	4.05E-02±5.60E-02	+	**2.04E-09±2.03E-09**	256.20±59.73	+	**195.45±39.24**
	F15	5.47E-01±4.35E-01	+	**2.82E-07±3.27E-07**	451.80±115.21	+	**244.50±44.50**
	F16	**1.06E+01±2.15E+01**	≈	1.27E+00±5.66E+00	575.50±28.21	+	**409.65±100.36**
	F17	5.39E+01±8.76E+00	≈	**4.50E+01±1.61E+01**	**600.00±0.00**	≈	**600.00±0.00**
	F18	**7.82E-01±1.81E+00**	−	1.55E+00±1.96E+00	**455.70±112.13**	−	538.25±93.69
	F19	**2.31E-01±3.31E-01**	≈	2.71E-01±7.16E-01	415.40±61.19	≈	**356.45±122.27**
	F20	1.08E+00±3.42E-01	+	**1.05E-01±3.23E-01**	570.80±46.20	+	**401.70±109.14**
Type 3	F21	5.65E-01±1.78E+00	+	**7.60E-08±5.64E-08**	536.50±41.89	+	**279.55±36.60**
	F22	5.24E+01±1.22E+01	≈	**4.78E+01±1.32E+01**	**600.00±0.00**	≈	**600.00±0.00**
	F23	6.39E-04±5.32E-04	≈	**2.38E-04±8.58E-05**	429.80±36.47	+	**363.15±76.32**
	F24	2.07E-02±3.27E-02	+	**1.18E-09±4.54E-10**	381.60±91.09	+	**185.15±37.79**
	F25	3.36E-01±2.59E-01	+	**1.24E-01±2.34E-01**	471.30±79.99	+	**229.45±32.75**
	F26	2.83E+00±7.16E+00	+	**9.70E-08±8.28E-08**	511.20±80.14	+	**330.80±62.07**
	F27	4.67E+01±9.75E+00	≈	**4.15E+01±1.78E+01**	**600.00±0.00**	≈	**600.00±0.00**
	F28	**9.55E-01±1.38E+00**	≈	9.76E-01±1.46E+00	526.70±76.02	≈	**475.15±117.83**
	F29	1.83E-01±1.46E-01	+	**3.74E-02±1.44E-01**	449.90±67.93	+	**273.10±83.76**
	F30	5.48E-01±3.94E-01	≈	**4.92E-01±6.18E-01**	491.60±77.34	+	**353.50±118.11**
+/−/≈		12/4/14			15/1/14		

TABLE S-XIV
RESULTS OF MISACO WITH VARYING N_{min} ON THE SIX SELECTED ARTIFICIAL TEST PROBLEMS.

N_{min}	$1*n_1$	$5*n_1$	$10*n_1$	$20*n_1$
	AOFV ± Std Dev			
F1	1.06E-07±4.49E-08	**6.21E-08±2.24E-08**	3.26E+00±5.38E+00	3.79E+00±2.89E+00
F6	1.49E-07±1.71E-07	**5.74E-08±2.50E-08**	3.79E+00±4.60E+00	8.05E+00±4.48E+00
F13	7.77E-01±1.52E-04	3.19E-01±1.29E+00	**2.48E-01±7.63E-01**	4.09E-02±3.71E-02
F18	2.65E+00±8.64E-01	1.55E+00±1.96E+00	**4.06E-01±3.28E-01**	7.61E-01±3.81E-01
F24	1.32E-06±1.92E-06	**9.90E-07±4.42E-06**	1.07E-02±2.44E-02	2.11E-02±3.19E-02
F29	9.30E-02±1.88E-01	**3.74E-02±1.44E-01**	2.25E-01±3.62E-01	2.94E-01±2.49E-01
	ASFEs ± Std Dev			
F1	311.40 ±52.30	**249.95 ±36.49**	567.40 ±33.05	588.10 ±35.91
F6	371.90 ±64.31	**269.40 ±56.18**	592.20 ±9.82	599.10 ±2.85
F13	305.90 ±50.10	**307.25 ±120.94**	465.80 ±59.35	510.90 ±39.68
F18	536.25 ±124.58	538.25 ±93.69	**483.50 ±61.50**	567.00 ±40.36
F24	272.10 ±80.85	**185.15 ±37.79**	268.20 ±58.16	288.50 ±53.52
F29	347.90 ±83.13	**273.10 ±83.76**	371.90 ±132.75	434.20 ±59.75

S-IV. Test Problems

A. The Constructed Artificial Test Problems

The characteristics of the constructed artificial test problems are summarized in the Table S-XV.

TABLE S-XV
CHARACTERISTICS OF THE 30 ARTIFICIAL TEST PROBLEMS

	Problem	n_1	n_2	l_j	L_i^{cn}	U_i^{cn}	Basic Function
Type 1	F1	8	2	5	-100	100	Sphere Function
	F2	8	2	5	-100	100	Rastrigin Function
	F3	8	2	5	-100	100	Alckey Function
	F4	8	2	5	-100	100	Elliposoid Function
	F5	8	2	5	-100	100	Griewank Function
	F6	8	2	10	-100	100	Sphere Function
	F7	8	2	10	-100	100	Rastrigin Function
	F8	8	2	10	-100	100	Alckey Function
	F9	8	2	10	-100	100	Elliposoid Function
	F10	8	2	10	-100	100	Griewank Function
Type 2	F11	2	8	5	-100	100	Sphere Function
	F12	2	8	5	-100	100	Rastrigin Function
	F13	2	8	5	-100	100	Alckey Function
	F14	2	8	5	-100	100	Elliposoid Function
	F15	2	8	5	-100	100	Griewank Function
	F16	2	8	10	-100	100	Sphere Function
	F17	2	8	10	-100	100	Rastrigin Function
	F18	2	8	10	-100	100	Alckey Function
	F19	2	8	10	-100	100	Elliposoid Function
	F20	2	8	10	-100	100	Griewank Function
Type 3	F21	5	5	5	-100	100	Sphere Function
	F22	5	5	5	-100	100	Rastrigin Function
	F23	5	5	5	-100	100	Alckey Function
	F24	5	5	5	-100	100	Elliposoid Function
	F25	5	5	5	-100	100	Griewank Function
	F26	5	5	10	-100	100	Sphere Function
	F27	5	5	10	-100	100	Rastrigin Function
	F28	5	5	10	-100	100	Alckey Function
	F29	5	5	10	-100	100	Elliposoid Function
	F30	5	5	10	-100	100	Griewank Function

The constructed 30 artificial test problems are as follows:

$$\begin{aligned}
\mathbf{F1}: \ min: \ & f(\mathbf{x}^{cn},\mathbf{x}^{ca}) = \sum_{i=1}^{10} z_i^2 \\
& \mathbf{z} = (z_1,\ldots,z_{10}) = \mathbf{z}'A \\
& z_i' = \begin{cases} x_i^{cn} - o_i, & i \in \{1,\ldots,8\} \\ x_{i-8}^{ca} - o_i, & i \in \{9,10\} \end{cases} \\
& \mathbf{o} = (o_1,\ldots,o_{10}) = (7.7624, -51.0984, -95.5110, -68.7425, 8.7344, 0.0577, -36.7734, 44.3837, 99.8131, -12.1793) \\
& \mathbf{v}_1 = \{v_1^1,\ldots,v_1^5\} = \{99.8131, 38.7794, 97.4385, 66.3214, 83.6572\} \\
& \mathbf{v}_2 = \{v_2^1,\ldots,v_2^5\} = \{-12.1793, -81.4490, 94.5925, -20.7460, -23.4447\} \\
& x_1^{cn},\ldots,x_8^{cn} \in [-100,100] \\
& x_j^{ca} \in \mathbf{v}_j, j \in \{1,2\}
\end{aligned}$$

where A is an orthogonal matrix. The optimal solution is: $\mathbf{x}_{best}^{cn} = (7.7624, -51.0984, -95.5110, -68.7425, 8.7344, 0.0577, -36.7734,$

44.3837) and $\mathbf{x}^{ca}_{best} = (99.8131, -12.1793)$, and $f(\mathbf{x}^{cn}_{best}, \mathbf{x}^{ca}_{best}) = 0$.

$$
\begin{aligned}
\textbf{F2}: \ min: \ \ & f(\mathbf{x}^{cn}, \mathbf{x}^{ca}) = \sum_{i=1}^{10} (z_i^2 - 10cos(2\pi z_i) + 10) \\
& \mathbf{z} = (z_1, \ldots, z_{10}) = 0.05 * \mathbf{z}' A \\
& z_i' = \begin{cases} x_i^{cn} - o_i, & i \in \{1, \ldots, 8\} \\ x_{i-8}^{ca} - o_i, & i \in \{9, 10\} \end{cases} \\
& \mathbf{o} = (o_1, \ldots, o_{10}) = (0.5876, -84.9703, -97.8543, 39.7223, 28.3686, 61.1286, 38.9558, -77.1346, 50.6776, 14.4813) \\
& \mathbf{v}_1 = \{v_1^1, \ldots, v_1^5\} = \{50.6776, -39.6234, -61.7100, 97.7223, 63.1775\} \\
& \mathbf{v}_2 = \{v_2^1, \ldots, v_2^5\} = \{14.4813, -97.4609, 92.2885, -3.8172, 83.2134\} \\
& x_1^{cn}, \ldots, x_8^{cn} \in [-100, 100] \\
& x_j^{ca} \in \mathbf{v}_j, j \in \{1, 2\}
\end{aligned}
$$

where A is an orthogonal matrix. The optimal solution is: $\mathbf{x}^{cn}_{best} = (0.5876, -84.9703, -97.8543, 39.7223, 28.3686, 61.1286, 38.9558, -77.1346)$ and $\mathbf{x}^{ca}_{best} = (50.6776, 14.4813)$, and $f(\mathbf{x}^{cn}_{best}, \mathbf{x}^{ca}_{best}) = 0$.

$$
\begin{aligned}
\textbf{F3}: \ min: \ \ & f(\mathbf{x}^{cn}, \mathbf{x}^{ca}) = 20 + e - 20\exp(-0.2\sqrt{\frac{1}{10}\sum_{i=1}^{10} z_i^2}) - \exp(\frac{1}{10}\sum_{i=1}^{10}\cos(2\pi z_i)) \\
& \mathbf{z} = (z_1, \ldots, z_{10}) = 0.32 * \mathbf{z}' A \\
& z_i' = \begin{cases} x_i^{cn} - o_i, & i \in \{1, \ldots, 8\} \\ x_{i-8}^{ca} - o_i, & i \in \{9, 10\} \end{cases} \\
& \mathbf{o} = (o_1, \ldots, o_{10}) = (5.3830, 0.1560, 47.8659, -17.0994, -32.5756, -29.2208, -32.7262, -43.5349, 1.9141, -36.7252) \\
& \mathbf{v}_1 = \{v_1^1, \ldots, v_1^5\} = \{1.9141, -12.6618, -3.5678, -18.1508, -7.8900\} \\
& \mathbf{v}_2 = \{v_2^1, \ldots, v_2^5\} = \{-36.7252, 27.8390, -53.1350, -21.6579, -27.2089\} \\
& x_1^{cn}, \ldots, x_8^{cn} \in [-100, 100] \\
& x_j^{ca} \in \mathbf{v}_j, j \in \{1, 2\}
\end{aligned}
$$

where A is an orthogonal matrix. The optimal solution is: $\mathbf{x}^{cn}_{best} = (5.3830, 0.1560, 47.8659, -17.0994, -32.5756, -29.2208, -32.7262, -43.5349)$ and $\mathbf{x}^{ca}_{best} = (1.9141, -36.7252)$, and $f(\mathbf{x}^{cn}_{best}, \mathbf{x}^{ca}_{best}) = 0$.

$$
\begin{aligned}
\textbf{F4}: \ min: \ \ & f(\mathbf{x}^{cn}, \mathbf{x}^{ca}) = \sum_{i=1}^{10} i x_i^2 \\
& \mathbf{z} = (z_1, \ldots, z_{10}) = 0.05 * \mathbf{z}' A \\
& z_i' = \begin{cases} x_i^{cn} - o_i, & i \in \{1, \ldots, 8\} \\ x_{i-8}^{ca} - o_i, & i \in \{9, 10\} \end{cases} \\
& \mathbf{o} = (o_1, \ldots, o_{10}) = (6.5706, -4.7415, 99.8101, -69.8675, 46.3398, -94.7804, -14.1227, -22.2035, 63.0211, -96.1546) \\
& \mathbf{v}_1 = \{v_1^1, \ldots, v_1^5\} = \{63.0211, 15.4585, -0.9510, 90.1246, 18.4635\} \\
& \mathbf{v}_2 = \{v_2^1, \ldots, v_2^5\} = \{-96.1546, -3.5673, -59.5396, 13.2944, -21.4157\} \\
& x_1^{cn}, \ldots, x_8^{cn} \in [-100, 100] \\
& x_j^{ca} \in \mathbf{v}_j, j \in \{1, 2\}
\end{aligned}
$$

where A is an orthogonal matrix. The optimal solution is: $\mathbf{x}^{cn}_{best} = (6.5706, -4.7415, 99.8101, -69.8675, 46.3398, -94.7804, -14.1227,$

$, -22.2035)$ and $\mathbf{x}^{ca}_{best} = (63.0211, -96.1546)$, and $f(\mathbf{x}^{cn}_{best}, \mathbf{x}^{ca}_{best}) = 0$.

$$
\begin{aligned}
\textbf{F5}: \ min: \ & f(\mathbf{x}^{cn}, \mathbf{x}^{ca}) = 1 + \sum_{i=1}^{10} \frac{z_i^2}{4000} - \prod_{i=1}^{10} \cos(\frac{z_i}{\sqrt{i}}) \\
& \mathbf{z} = (z_1, \ldots, z_{10}) = 6 * \mathbf{z}'A \\
& z_i' = \begin{cases} x_i^{cn} - o_i, & i \in \{1, \ldots, 8\} \\ x_{i-8}^{ca} - o_i, & i \in \{9, 10\} \end{cases} \\
& \mathbf{o} = (o_1, \ldots, o_{10}) = (84.9982, 65.1519, -12.0721, -10.4880, 2.4412, -86.8010, -32.6663, -39.7689, 39.6034, 52.0954) \\
& \mathbf{v}_1 = \{v_1^1, \ldots, v_1^5\} = \{39.6034, -98.0900, 66.9081, 5.6974, -52.6892\} \\
& \mathbf{v}_2 = \{v_2^1, \ldots, v_2^5\} = \{52.0954, -95.0943, 60.2757, -59.7487, -89.1344\} \\
& x_1^{cn}, \ldots, x_5^{cn} \in [-100, 100] \\
& x_j^{ca} \in \mathbf{v}_j, j \in \{1, 2\}
\end{aligned}
$$

where A is an orthogonal matrix. The optimal solution is: $\mathbf{x}^{cn}_{best} = (84.9982, 65.1519, -12.0721, -10.4880, 2.4412, -86.8010, -32.6663, -39.7689)$ and $\mathbf{x}^{ca}_{best} = (39.6034, 52.0954)$, and $f(\mathbf{x}^{cn}_{best}, \mathbf{x}^{ca}_{best}) = 0$.

$$
\begin{aligned}
\textbf{F6}: \ min: \ & f(\mathbf{x}^{cn}, \mathbf{x}^{ca}) = \sum_{i=1}^{10} z_i^2 \\
& \mathbf{z} = (z_1, \ldots, z_{10}) = \mathbf{z}'A \\
& z_i' = \begin{cases} x_i^{cn} - o_i, & i \in \{1, \ldots, 8\} \\ x_{i-8}^{ca} - o_i, & i \in \{9, 10\} \end{cases} \\
& \mathbf{o} = (o_1, \ldots, o_{10}) = (7.7624, -51.0984, -95.5110, -68.7425, 8.7344, 0.0577, -36.7734, 44.3837, 99.8131, -12.1793) \\
& \mathbf{v}_1 = \{v_1^1, \ldots, v_1^{10}\} = \{99.8131, 38.7794, 97.4385, 66.3214, 83.6572, 64.3900, 6.2714, 4.7893, 42.9978, -46.2021\} \\
& \mathbf{v}_2 = \{v_2^1, \ldots, v_2^{10}\} = \{-12.1793, -81.4490, 94.5925, -20.7460, -23.4447, -37.0443, -33.7724, -78.8025, 69.8750, \\
& \qquad 70.8563\} \\
& x_1^{cn}, \ldots, x_8^{cn} \in [-100, 100] \\
& x_j^{ca} \in \mathbf{v}_j, j \in \{1, 2\}
\end{aligned}
$$

where A is an orthogonal matrix. The optimal solution is: $\mathbf{x}^{cn}_{best} = (7.7624, -51.0984, -95.5110, -68.7425, 8.7344, 0.0577, -36.7734, 44.3837)$ and $\mathbf{x}^{ca}_{best} = (99.8131, -12.1793)$, and $f(\mathbf{x}^{cn}_{best}, \mathbf{x}^{ca}_{best}) = 0$.

$$
\begin{aligned}
\textbf{F7}: \ min: \ & f(\mathbf{x}^{cn}, \mathbf{x}^{ca}) = \sum_{i=1}^{10} (z_i^2 - 10cos(2\pi z_i) + 10) \\
& \mathbf{z} = (z_1, \ldots, z_{10}) = 0.05 * \mathbf{z}'A \\
& z_i' = \begin{cases} x_i^{cn} - o_i, & i \in \{1, \ldots, 8\} \\ x_{i-8}^{ca} - o_i, & i \in \{9, 10\} \end{cases} \\
& \mathbf{o} = (o_1, \ldots, o_{10}) = (0.5876, -84.9703, -97.8543, 39.7223, 28.3686, 61.1286, 38.9558, -77.1346, 50.6776, 14.4813) \\
& \mathbf{v}_1 = \{v_1^1, \ldots, v_1^{10}\} = \{50.6776, -39.6234, -61.7100, 97.7223, 63.1775, -12.0348, -21.6271, -12.3744, 67.1491, \\
& \qquad -19.0775\} \\
& \mathbf{v}_2 = \{v_2^1, \ldots, v_2^{10}\} = \{14.4813, -97.4609, 92.2885, -3.8172, 83.2134, -89.4358, 10.1637, -86.6364, -64.1289, 6.0189\} \\
& x_1^{cn}, \ldots, x_8^{cn} \in [-100, 100] \\
& x_j^{ca} \in \mathbf{v}_j, j \in \{1, 2\}
\end{aligned}
$$

where A is an orthogonal matrix. The optimal solution is: $\mathbf{x}^{cn}_{best} = (0.5876, -84.9703, -97.8543, 39.7223, 28.3686, 61.1286, 38.9558,$

$-77.1346)$ and $\mathbf{x}_{best}^{ca} = (50.6776, 14.4813)$, and $f(\mathbf{x}_{best}^{cn}, \mathbf{x}_{best}^{ca}) = 0$.

$$
\begin{aligned}
\textbf{F8}: \ min: \ & f(\mathbf{x}^{cn}, \mathbf{x}^{ca}) = 20 + e - 20\exp(-0.2\sqrt{\frac{1}{10}\sum_{i=1}^{10} z_i^2}) - \exp(\frac{1}{10}\sum_{i=1}^{10}\cos(2\pi z_i)) \\
& \mathbf{z} = (z_1, \ldots, z_{10}) = 0.32 * \mathbf{z}'A \\
& z_i' = \begin{cases} x_i^{cn} - o_i, & i \in \{1, \ldots, 8\} \\ x_{i-8}^{ca} - o_i, & i \in \{9, 10\} \end{cases} \\
& \mathbf{o} = (o_1, \ldots, o_{10}) = (5.3830, 0.1560, 47.8659, -17.0994, -32.5756, -29.2208, -32.7262, -43.5349, 1.9141, -36.7252) \\
& \mathbf{v}_1 = \{v_1^1, \ldots, v_1^{10}\} = \{1.9141, -12.6618, -3.5678, -18.1508, -7.8900, 1.7955, -45.1022, -14.7915, -47.5095, \\
& \qquad 57.2121\} \\
& \mathbf{v}_2 = \{v_2^1, \ldots, v_2^{10}\} = \{-36.7252, 27.8390, -53.1350, -21.6579, -27.2089, -58.0376, 19.1243, 2.8412, -17.4512, \\
& \qquad -58.3012\} \\
& x_1^{cn}, \ldots, x_8^{cn} \in [-100, 100] \\
& x_j^{ca} \in \mathbf{v}_j, j \in \{1, 2\}
\end{aligned}
$$

where A is an orthogonal matrix. The optimal solution is: $\mathbf{x}_{best}^{cn} = (5.3830, 0.1560, 47.8659, -17.0994, -32.5756, -29.2208, -32.7262, -43.5349)$ and $\mathbf{x}_{best}^{ca} = (1.9141, -36.7252)$, and $f(\mathbf{x}_{best}^{cn}, \mathbf{x}_{best}^{ca}) = 0$.

$$
\begin{aligned}
\textbf{F9}: \ min: \ & f(\mathbf{x}^{cn}, \mathbf{x}^{ca}) = \sum_{i=1}^{10} i x_i^2 \\
& \mathbf{z} = (z_1, \ldots, z_{10}) = 0.05 * \mathbf{z}'A \\
& z_i' = \begin{cases} x_i^{cn} - o_i, & i \in \{1, \ldots, 8\} \\ x_{i-8}^{ca} - o_i, & i \in \{9, 10\} \end{cases} \\
& \mathbf{o} = (o_1, \ldots, o_{10}) = (6.5706, -4.7415, 99.8101, -69.8675, 46.3398, -94.7804, -14.1227, -22.2035, 63.0211, -96.1546) \\
& \mathbf{v}_1 = \{v_1^1, \ldots, v_1^{10}\} = \{63.0211, 15.4585, -0.9510, 90.1246, 18.4635, -17.3490, 96.5306, -14.2523, -37.9036, 58.6272\} \\
& \mathbf{v}_2 = \{v_2^1, \ldots, v_2^{10}\} = \{-96.1546, -3.5673, -59.5396, 13.2944, -21.4157, 25.7777, 0.5632, 73.3501, -29.0539, -79.8143\} \\
& x_1^{cn}, \ldots, x_8^{cn} \in [-100, 100] \\
& x_j^{ca} \in \mathbf{v}_j, j \in \{1, 2\}
\end{aligned}
$$

where A is an orthogonal matrix. The optimal solution is: $\mathbf{x}_{best}^{cn} = (6.5706, -4.7415, 99.8101, -69.8675, 46.3398, -94.7804, -14.1227, -22.2035)$ and $\mathbf{x}_{best}^{ca} = (63.0211, -96.1546)$, and $f(\mathbf{x}_{best}^{cn}, \mathbf{x}_{best}^{ca}) = 0$.

$$
\begin{aligned}
\textbf{F10}: \ min: \ & f(\mathbf{x}^{cn}, \mathbf{x}^{ca}) = 1 + \sum_{i=1}^{10} \frac{z_i^2}{4000} - \prod_{i=1}^{10} \cos(\frac{z_i}{\sqrt{i}}) \\
& \mathbf{z} = (z_1, \ldots, z_{10}) = 6 * \mathbf{z}'A \\
& z_i' = \begin{cases} x_i^{cn} - o_i, & i \in \{1, \ldots, 8\} \\ x_{i-8}^{ca} - o_i, & i \in \{9, 10\} \end{cases} \\
& \mathbf{o} = (o_1, \ldots, o_{10}) = (84.9982, 65.1519, -12.0721, -10.4880, 2.4412, -86.8010, -32.6663, -39.7689, 39.6034, 52.0954) \\
& \mathbf{v}_1 = \{v_1^1, \ldots, v_1^{10}\} = \{39.6034, -98.0900, 66.9081, 5.6974, -52.6892, -22.8678, -21.4237, -70.5337, 8.6142, -89.3348\} \\
& \mathbf{v}_2 = \{v_2^1, \ldots, v_2^{10}\} = \{52.0954, -95.0943, 60.2757, -59.7487, -89.1344, 70.3986, -55.3637, 17.6185, -72.3865, \\
& \qquad -10.6520\} \\
& x_1^{cn}, \ldots, x_8^{cn} \in [-100, 100] \\
& x_j^{ca} \in \mathbf{v}_j, j \in \{1, 2\}
\end{aligned}
$$

where A is an orthogonal matrix. The optimal solution is: $\mathbf{x}_{best}^{cn} = (84.9982, 65.1519, -12.0721, -10.4880, 2.4412, -86.8010, -32.6663,$

$-39.7689)$ and $\mathbf{x}^{ca}_{best} = (39.6034, 52.0954)$, and $f(\mathbf{x}^{cn}_{best}, \mathbf{x}^{ca}_{best}) = 0$.

$$
\begin{aligned}
\textbf{F11}:\ min:\ \ & f(\mathbf{x}^{cn}, \mathbf{x}^{ca}) = \sum_{i=1}^{10} z_i^2 \\
& \mathbf{z} = (z_1, \ldots, z_{10}) = \mathbf{z}' A \\
& z'_i = \begin{cases} x^{cn}_i - o_i, & i \in \{1,2\} \\ x^{ca}_{i-2} - o_i, & i \in \{3, \ldots, 10\} \end{cases} \\
& \mathbf{o} = (o_1, \ldots, o_{10}) = (7.7624, -51.0984, -95.5110, -68.7425, 8.7344, 0.0577, -36.7734, 44.3837, 99.8131, -12.1793) \\
& \mathbf{v}_1 = \{v_1^1, \ldots, v_1^5\} = \{-95.5110, 10.9166, -86.3500, 6.3552, -52.8390\} \\
& \mathbf{v}_2 = \{v_2^1, \ldots, v_2^5\} = \{-68.7425, 2.4009, -26.8628, 52.9171, -94.4758\} \\
& \mathbf{v}_3 = \{v_3^1, \ldots, v_3^5\} = \{8.7344, 2.0220, 1.2974, -37.0691, -79.2651\} \\
& \mathbf{v}_4 = \{v_4^1, \ldots, v_4^5\} = \{0.0577, -66.8891, -24.5506, -96.2061, 45.4579\} \\
& \mathbf{v}_5 = \{v_5^1, \ldots, v_5^5\} = \{-36.7734, 11.1348, 40.9187, -32.3377, 62.3757\} \\
& \mathbf{v}_6 = \{v_6^1, \ldots, v_6^5\} = \{44.3837, -84.2635, -31.8857, -99.0299, 23.2041\} \\
& \mathbf{v}_7 = \{v_7^1, \ldots, v_7^5\} = \{99.8131, 38.7794, 97.4385, 66.3214, 83.6572\} \\
& \mathbf{v}_8 = \{v_8^1, \ldots, v_8^5\} = \{-12.1793, -81.4490, 94.5925, -20.7460, -23.4447\} \\
& x_1^{cn}, x_2^{cn} \in [-100, 100] \\
& x_j^{ca} \in \mathbf{v}_j, j \in \{1,2,3,4,5,6,7,8\}
\end{aligned}
$$

where A is an orthogonal matrix. The optimal solution is: $\mathbf{x}^{cn}_{best} = (7.7624, -51.0984)$ and $\mathbf{x}^{ca}_{best} = (-95.5110, -68.7425, 8.7344,$ $0.0577, -36.7734, 44.3837, 99.8131, -12.1793)$, and $f(\mathbf{x}^{cn}_{best}, \mathbf{x}^{ca}_{best}) = 0$.

$$
\begin{aligned}
\textbf{F12}:\ min:\ \ & f(\mathbf{x}^{cn}, \mathbf{x}^{ca}) = \sum_{i=1}^{10} (z_i^2 - 10cos(2\pi z_i) + 10) \\
& \mathbf{z} = (z_1, \ldots, z_{10}) = 0.05 * \mathbf{z}' A \\
& z'_i = \begin{cases} x^{cn}_i - o_i, & i \in \{1,2\} \\ x^{ca}_{i-2} - o_i, & i \in \{3, \ldots, 10\} \end{cases} \\
& \mathbf{o} = (o_1, \ldots, o_{10}) = (0.5876, -84.9703, -97.8543, 39.7223, 28.3686, 61.1286, 38.9558, -77.1346, 50.6776, 14.4813) \\
& \mathbf{v}_1 = \{v_1^1, \ldots, v_1^5\} = \{-97.8543, -98.8581, -93.8085, -78.8370, 81.9188\} \\
& \mathbf{v}_2 = \{v_2^1, \ldots, v_2^5\} = \{39.7223, -99.8416, -4.6454, 74.7200, -80.8983\} \\
& \mathbf{v}_3 = \{v_3^1, \ldots, v_3^5\} = \{28.3686, -19.5009, -96.7178, 9.0181, 58.1322\} \\
& \mathbf{v}_4 = \{v_4^1, \ldots, v_4^5\} = \{61.1286, 4.1286, -3.9445, -15.1705, -7.9428\} \\
& \mathbf{v}_5 = \{v_5^1, \ldots, v_5^5\} = \{38.9558, 37.3101, 83.0975, -98.6905, -75.3426\} \\
& \mathbf{v}_6 = \{v_6^1, \ldots, v_6^5\} = \{-77.1346, -2.0340, 70.8802, 51.3176, -24.3460\} \\
& \mathbf{v}_7 = \{v_7^1, \ldots, v_7^5\} = \{50.6776, -39.6234, -61.7100, 97.7223, 63.1775\} \\
& \mathbf{v}_8 = \{v_8^1, \ldots, v_8^5\} = \{14.4813, -97.4609, 92.2885, -3.8172, 83.2134\} \\
& x_1^{cn}, x_2^{cn} \in [-100, 100] \\
& x_j^{ca} \in \mathbf{v}_j, j \in \{1,2,3,4,5,6,7,8\}
\end{aligned}
$$

where A is an orthogonal matrix. The optimal solution is: $\mathbf{x}^{cn}_{best} = (0.5876, -84.9703)$ and $\mathbf{x}^{ca}_{best} = (-97.8543, 39.7223, 28.3686,$

$61.1286, 38.9558, -77.1346, 50.6776, 14.4813)$, and $f(\mathbf{x}^{cn}_{best}, \mathbf{x}^{ca}_{best}) = 0$.

$$
\begin{aligned}
\mathbf{F13}: \ min: \ & f(\mathbf{x}^{cn}, \mathbf{x}^{ca}) = 20 + e - 20\exp(-0.2\sqrt{\frac{1}{10}\sum_{i=1}^{10} z_i^2}) - \exp(\frac{1}{10}\sum_{i=1}^{10}\cos(2\pi z_i)) \\
& \mathbf{z} = (z_1, \ldots, z_{10}) = 0.32 * \mathbf{z}' A \\
& z'_i = \begin{cases} x^{cn}_i - o_i, & i \in \{1,2\} \\ x^{ca}_{i-2} - o_i, & i \in \{3,\ldots,10\} \end{cases} \\
& \mathbf{o} = (o_1, \ldots, o_{10}) = (5.3830, 0.1560, 47.8659, -17.0994, -32.5756, -29.2208, -32.7262, -43.5349, 1.9141, -36.7252) \\
& \mathbf{v}_1 = \{v_1^1, \ldots, v_1^5\} = \{47.8659, -50.2789, -51.4218, -5.7807, 59.7290\} \\
& \mathbf{v}_2 = \{v_2^1, \ldots, v_2^5\} = \{-17.0994, 17.6311, -52.2380, -37.5205, -50.8626\} \\
& \mathbf{v}_3 = \{v_3^1, \ldots, v_3^5\} = \{-32.5756, 27.7492, -33.1525, 55.2097, -38.2367\} \\
& \mathbf{v}_4 = \{v_4^1, \ldots, v_4^5\} = \{-29.2208, -14.6371, 31.1216, -14.0404, 23.6011\} \\
& \mathbf{v}_5 = \{v_5^1, \ldots, v_5^5\} = \{-32.7262, 53.3853, -56.2406, 51.0570, 21.1575\} \\
& \mathbf{v}_6 = \{v_6^1, \ldots, v_6^5\} = \{-43.5349, -59.4140, -49.4245, -9.3890, -56.9489\} \\
& \mathbf{v}_7 = \{v_7^1, \ldots, v_7^5\} = \{1.9141, -12.6618, -3.5678, -18.1508, -7.8900\} \\
& \mathbf{v}_8 = \{v_8^1, \ldots, v_8^5\} = \{-36.7252, 27.8390, -53.1350, -21.6579, -27.2089\} \\
& x^{cn}_1, x^{cn}_2 \in [-100, 100] \\
& x^{ca}_j \in \mathbf{v}_j, j \in \{1,2,3,4,5,6,7,8\}
\end{aligned}
$$

where A is an orthogonal matrix. The optimal solution is: $\mathbf{x}^{cn}_{best} = (5.3830, 0.1560)$ and $\mathbf{x}^{ca}_{best} = (47.8659, -17.0994, -32.5756, -29.2208, -32.7262, -43.5349, 1.9141, -36.7252)$, and $f(\mathbf{x}^{cn}_{best}, \mathbf{x}^{ca}_{best}) = 0$.

$$
\begin{aligned}
\mathbf{F14}: \ min: \ & f(\mathbf{x}^{cn}, \mathbf{x}^{ca}) = \sum_{i=1}^{10} i x_i^2 \\
& \mathbf{z} = (z_1, \ldots, z_{10}) = 0.05 * \mathbf{z}' A \\
& z'_i = \begin{cases} x^{cn}_i - o_i, & i \in \{1,2\} \\ x^{ca}_{i-2} - o_i, & i \in \{3,\ldots,10\} \end{cases} \\
& \mathbf{o} = (o_1, \ldots, o_{10}) = (6.5706, -4.7415, 99.8101, -69.8675, 46.3398, -94.7804, -14.1227, -22.2035, 63.0211, -96.1546) \\
& \mathbf{v}_1 = \{v_1^1, \ldots, v_1^5\} = \{99.8101, -31.6099, 89.6818, 56.8515, -88.4346\} \\
& \mathbf{v}_2 = \{v_2^1, \ldots, v_2^5\} = \{-69.8675, 40.6365, -76.1113, 65.0137, -93.6473\} \\
& \mathbf{v}_3 = \{v_3^1, \ldots, v_3^5\} = \{46.3398, 57.3869, -59.6658, 54.2175, -66.4293\} \\
& \mathbf{v}_4 = \{v_4^1, \ldots, v_4^5\} = \{-94.7804, 90.0091, -12.3052, 27.1707, 89.5608\} \\
& \mathbf{v}_5 = \{v_5^1, \ldots, v_5^5\} = \{-14.1227, 70.3685, -3.5821, -57.3306, -21.0553\} \\
& \mathbf{v}_6 = \{v_6^1, \ldots, v_6^5\} = \{-22.2035, -94.6947, -28.3114, 79.0958, -32.9711\} \\
& \mathbf{v}_7 = \{v_7^1, \ldots, v_7^5\} = \{63.0211, 15.4585, -0.9510, 90.1246, 18.4635\} \\
& \mathbf{v}_8 = \{v_8^1, \ldots, v_8^5\} = \{-96.1546, -3.5673, -59.5396, 13.2944, -21.4157\} \\
& x^{cn}_1, x^{cn}_2 \in [-100, 100] \\
& x^{ca}_j \in \mathbf{v}_j, j \in \{1,2,3,4,5,6,7,8\}
\end{aligned}
$$

where A is an orthogonal matrix. The optimal solution is: $\mathbf{x}^{cn}_{best} = (6.5706, -4.7415)$ and $\mathbf{x}^{ca}_{best} = (99.8101, -69.8675, 46.3398, -94.7804,$

$-14.1227, -22.2035, 63.0211, -96.1546)$, and $f(\mathbf{x}^{cn}_{best}, \mathbf{x}^{ca}_{best}) = 0$.

$$
\begin{aligned}
\textbf{F15}: \ min: \ & f(\mathbf{x}^{cn}, \mathbf{x}^{ca}) = 1 + \sum_{i=1}^{10} \frac{z_i^2}{4000} - \prod_{i=1}^{10} \cos(\frac{z_i}{\sqrt{i}}) \\
& \mathbf{z} = (z_1, \ldots, z_{10}) = 6 * \mathbf{z}' A \\
& z'_i = \begin{cases} x^{cn}_i - o_i, & i \in \{1,2\} \\ x^{ca}_{i-2} - o_i, & i \in \{3,\ldots,10\} \end{cases} \\
& \mathbf{o} = (o_1, \ldots, o_{10}) = (84.9982, 65.1519, -12.0721, -10.4880, 2.4412, -86.8010, -32.6664, -39.7689, 39.6034, 52.0954) \\
& \mathbf{v}_1 = \{v_1^1, \ldots, v_1^5\} = \{-12.0721, 2.9979, -19.0989, -67.9707, -5.3256\} \\
& \mathbf{v}_2 = \{v_2^1, \ldots, v_2^5\} = \{-10.4880, 49.2424, 64.1867, 5.7290, -95.5992\} \\
& \mathbf{v}_3 = \{v_3^1, \ldots, v_3^5\} = \{2.4412, 63.1647, -11.6669, -31.9233, -87.5237\} \\
& \mathbf{v}_4 = \{v_4^1, \ldots, v_4^5\} = \{-86.8010, 58.9903, -71.2887, 55.7150, 59.1220\} \\
& \mathbf{v}_5 = \{v_5^1, \ldots, v_5^5\} = \{-32.6664, 14.3942, 19.6051, 23.8658, 92.7302\} \\
& \mathbf{v}_6 = \{v_6^1, \ldots, v_6^5\} = \{-39.7689, 51.2260, 95.1572, -52.9366, 81.0941\} \\
& \mathbf{v}_7 = \{v_7^1, \ldots, v_7^5\} = \{39.6034, -98.0900, 66.9081, 5.6974, -52.6892\} \\
& \mathbf{v}_8 = \{v_8^1, \ldots, v_8^5\} = \{52.0954, -95.0943, 60.2757, -59.7487, -89.1344\} \\
& x_1^{cn}, x_2^{cn} \in [-100, 100] \\
& x_j^{ca} \in \mathbf{v}_j, j \in \{1,2,3,4,5,6,7,8\}
\end{aligned}
$$

where A is an orthogonal matrix. The optimal solution is: $\mathbf{x}^{cn}_{best} = (84.9982, 65.1519)$ and $\mathbf{x}^{ca}_{best} = (-12.0721, -10.4880, 2.4412, -86.8010, -32.6664, -39.7689, 39.6034, 52.0954)$, and $f(\mathbf{x}^{cn}_{best}, \mathbf{x}^{ca}_{best}) = 0$.

$$
\begin{aligned}
\textbf{F16}: \ min: \ & f(\mathbf{x}^{cn}, \mathbf{x}^{ca}) = \sum_{i=1}^{10} z_i^2 \\
& \mathbf{z} = (z_1, \ldots, z_{10}) = \mathbf{z}' A \\
& z'_i = \begin{cases} x^{cn}_i - o_i, & i \in \{1,2\} \\ x^{ca}_{i-2} - o_i, & i \in \{3,\ldots,10\} \end{cases} \\
& \mathbf{o} = (o_1, \ldots, o_{10}) = (7.7624, -51.0984, -95.5110, -68.7425, 8.7344, 0.0577, -36.7734, 44.3837, 99.8131, -12.1793) \\
& \mathbf{v}_1 = \{v_1^1, \ldots, v_1^{10}\} = \{-95.5110, 10.9166, -86.3500, 6.3552, -52.8390, 30.5276, 77.9978, -14.5499, -53.7453, 93.4961\} \\
& \mathbf{v}_2 = \{v_2^1, \ldots, v_2^{10}\} = \{-68.7425, 2.4009, -26.8628, 52.9171, -94.4758, -19.8521, 18.5924, 16.7370, -83.7091, -33.1471\} \\
& \mathbf{v}_3 = \{v_3^1, \ldots, v_3^{10}\} = \{8.7344, 2.0220, 1.2974, -37.0691, -79.2651, -13.4857, 91.8744, 41.5887, 54.6449, -53.6206\} \\
& \mathbf{v}_4 = \{v_4^1, \ldots, v_4^{10}\} = \{0.0577, -66.8891, -24.5506, -96.2061, 45.4579, 18.1319, 58.3241, 71.9285, -75.8105, -22.9862\} \\
& \mathbf{v}_5 = \{v_5^1, \ldots, v_5^{10}\} = \{-36.7734, 11.1348, 40.9187, -32.3377, 62.3757, 24.4644, 86.2232, 90.6468, -99.4558, 89.3221\} \\
& \mathbf{v}_6 = \{v_6^1, \ldots, v_6^{10}\} = \{44.3837, -84.2635, -31.8857, -99.0299, 23.2041, 13.1996, -43.5760, 26.1334, -6.6750, -22.8134\} \\
& \mathbf{v}_7 = \{v_7^1, \ldots, v_7^{10}\} = \{99.8131, 38.7794, 97.4385, 66.3214, 83.6572, 64.3900, 6.2714, 4.7893, 42.9978, -46.2021\} \\
& \mathbf{v}_8 = \{v_8^1, \ldots, v_8^{10}\} = \{-12.1793, -81.4490, 94.5925, -20.7460, -23.4447, -37.0443, -33.7724, -78.8025, 69.8750, \\
& \qquad 70.8563\} \\
& x_1^{cn}, \ldots, x_2^{cn} \in [-100, 100] \\
& x_j^{ca} \in \mathbf{v}_j, j \in \{1,2,3,4,5,6,7,8\}
\end{aligned}
$$

where A is an orthogonal matrix. The optimal solution is: $\mathbf{x}^{cn}_{best} = (7.7624, -51.0984)$ and $\mathbf{x}^{ca}_{best} = (-95.5110, -68.7425, 8.7344,$

$0.0577, -36.7734, 44.3837, 99.8131, -12.1793)$, and $f(\mathbf{x}^{cn}_{best}, \mathbf{x}^{ca}_{best}) = 0$.

$$
\begin{aligned}
\textbf{F17}:\ min:\ \ & f(\mathbf{x}^{cn}, \mathbf{x}^{ca}) = \sum_{i=1}^{10}(z_i^2 - 10cos(2\pi z_i) + 10) \\
& \mathbf{z} = (z_1, \ldots, z_{10}) = 0.05 * \mathbf{z}' A \\
& z'_i = \begin{cases} x^{cn}_i - o_i, & i \in \{1, \ldots, 2\} \\ x^{ca}_{i-2} - o_i, & i \in \{3, \ldots, 10\} \end{cases} \\
& \mathbf{o} = (o_1, \ldots, o_{10}) = (0.5876, -84.9703, -97.8543, 39.7223, 28.3686, 61.1286, 38.9558, -77.1346, 50.6776, 14.4813) \\
& \mathbf{v}_1 = \{v_1^1, \ldots, v_1^{10}\} = \{-97.8543, -98.8581, -93.8085, -78.8370, 81.9188, -2.9181, -50.9363, 31.9732, 69.0271, 85.5965\} \\
& \mathbf{v}_2 = \{v_2^1, \ldots, v_2^{10}\} = \{39.7223, -99.8416, -4.6454, 74.7200, -80.8983, -54.2256, 73.2056, -29.1147, -11.7113, 16.2659\} \\
& \mathbf{v}_3 = \{v_3^1, \ldots, v_3^{10}\} = \{28.3686, -19.5009, -96.7178, 9.0181, 58.1322, -8.9950, 99.3704, 28.2749, 60.8476, -53.6360\} \\
& \mathbf{v}_4 = \{v_4^1, \ldots, v_4^{10}\} = \{61.1286, 4.1286, -3.9445, -15.1705, -7.9428, -51.8438, 76.1710, -37.0110, -34.0411, -61.8862\} \\
& \mathbf{v}_5 = \{v_5^1, \ldots, v_5^{10}\} = \{38.9558, 37.3101, 83.0975, -98.6905, -75.3426, 93.3490, 52.6871, -77.6949, 3.4488, -14.2671\} \\
& \mathbf{v}_6 = \{v_6^1, \ldots, v_6^{10}\} = \{-77.1346, -2.0340, 70.8802, 51.3176, -24.3460, 63.8007, 93.8858, 11.8351, 57.5630, -0.9117\} \\
& \mathbf{v}_7 = \{v_7^1, \ldots, v_7^{10}\} = \{50.6776, -39.6234, -61.7100, 97.7223, 63.1775, -12.0348, -21.6271, -12.3744, 67.1491, \\
& \qquad -19.0775\} \\
& \mathbf{v}_8 = \{v_8^1, \ldots, v_8^{10}\} = (14.4813, -97.4609, 92.2885, -3.8172, 83.2134, -89.4358, 10.1637, -86.6364, -64.1289, 6.0189\} \\
& x^{cn}_1, \ldots, x^{cn}_2 \in [-100, 100] \\
& x^{ca}_j \in \mathbf{v}_j, j \in \{1, 2, 3, 4, 5, 6, 7, 8\}
\end{aligned}
$$

where A is an orthogonal matrix. The optimal solution is: $\mathbf{x}^{cn}_{best} = (0.5876, -84.9703)$ and $\mathbf{x}^{ca}_{best} = (-97.8543, 39.7223, 28.3686, 61.1286, 38.9558, -77.1346, 50.6776, 14.4813)$, and $f(\mathbf{x}^{cn}_{best}, \mathbf{x}^{ca}_{best}) = 0$.

$$
\begin{aligned}
\textbf{F18}:\ min:\ \ & f(\mathbf{x}^{cn}, \mathbf{x}^{ca}) = 20 + e - 20\exp(-0.2\sqrt{\frac{1}{10}\sum_{i=1}^{10} z_i^2}) - \exp(\frac{1}{10}\sum_{i=1}^{10}\cos(2\pi z_i)) \\
& \mathbf{z} = (z_1, \ldots, z_{10}) = 0.32 * \mathbf{z}' A \\
& z'_i = \begin{cases} x^{cn}_i - o_i, & i \in \{1, \ldots, 2\} \\ x^{ca}_{i-2} - o_i, & i \in \{3, \ldots, 10\} \end{cases} \\
& \mathbf{o} = (o_1, \ldots, o_{10}) = (5.3830, 0.1560, 47.8659, -17.0994, -32.5756, -29.2208, -32.7262, -43.5349, 1.9141, -36.7252) \\
& \mathbf{v}_1 = \{v_1^1, \ldots, v_1^{10}\} = \{47.8659, -50.2789, -51.4218, -5.7807, 59.7290, -16.4921, 4.6304, -51.4052, -56.5858, -1.4768\} \\
& \mathbf{v}_2 = \{v_2^1, \ldots, v_2^{10}\} = \{-17.0994, 17.6311, -52.2380, -37.5205, -50.8626, -18.6013, -3.3562, -46.1564, 23.0041, \\
& \qquad -24.2366\} \\
& \mathbf{v}_3 = \{v_3^1, \ldots, v_3^{10}\} = \{-32.5756, 27.7492, -33.1525, 55.2097, -38.2367, -34.8929, 54.6377, -45.0192, 26.5879, \\
& \qquad -53.1159\} \\
& \mathbf{v}_4 = \{v_4^1, \ldots, v_4^{10}\} = \{-29.2208, -14.6371, 31.1216, -14.0404, 23.6011, -27.5998, 50.6179, 26.2418, 43.5885, 15.1259\} \\
& \mathbf{v}_5 = \{v_5^1, \ldots, v_5^{10}\} = \{-32.7262, 53.3853, -56.2406, 51.0570, 21.1575, 20.4656, -28.2398, -13.7042, -18.0062, 10.0906\} \\
& \mathbf{v}_6 = \{v_6^1, \ldots, v_6^{10}\} = \{-43.5349, -59.4140, -49.4245, -9.3890, -56.9489, -50.0486, -53.9074, -58.2566, 17.3949, \\
& \qquad -27.1704\} \\
& \mathbf{v}_7 = \{v_7^1, \ldots, v_7^{10}\} = \{1.9141, -12.6618, -3.5678, -18.1508, -7.8900, 1.7955, -45.1022, -14.7915, -47.5095, -57.2121\} \\
& \mathbf{v}_8 = \{v_8^1, \ldots, v_8^{10}\} = \{-36.7252, 27.8390, -53.1350, -21.6579, -27.2089, -58.0376, 19.1243, 2.8412, -17.4512, \\
& \qquad -58.3012\} \\
& x^{cn}_1, \ldots, x^{cn}_5 \in [-100, 100] \\
& x^{ca}_j \in \mathbf{v}_j, j \in \{1, 2, 3, 4, 5, 6, 7, 8\}
\end{aligned}
$$

where A is an orthogonal matrix. The optimal solution is: $\mathbf{x}^{cn}_{best} = (5.3830, 0.1560)$ and $\mathbf{x}^{ca}_{best} = (47.8659, -17.0994, -32.5756, -29.2208,$

$-32.7262, -43.5349, 1.9141, -36.7252)$, and $f(\mathbf{x}^{cn}_{best}, \mathbf{x}^{ca}_{best}) = 0$.

$$
\begin{aligned}
\mathbf{F19}: \ min: \ & f(\mathbf{x}^{cn}, \mathbf{x}^{ca}) = \sum_{i=1}^{10} i x_i^2 \\
& \mathbf{z} = (z_1, \ldots, z_{10}) = 0.05 * \mathbf{z}' A \\
& z'_i = \begin{cases} x^{cn}_i - o_i, & i \in \{1, \ldots, 2\} \\ x^{ca}_{i-2} - o_i, & i \in \{3, \ldots, 10\} \end{cases} \\
& \mathbf{o} = (o_1, \ldots, o_{10}) = (6.5706, -4.7415, 99.8101, -69.8675, 46.3398, -94.7804, -14.1227, -22.2035, 63.0211, -96.1546) \\
& \mathbf{v}_1 = \{v_1^1, \ldots, v_1^{10}\} = \{99.8101, -31.6099, 89.6818, 56.8515, -88.4346, -57.0659, -31.5381, -49.4641, 25.9274, 9.5564\} \\
& \mathbf{v}_2 = \{v_2^1, \ldots, v_2^{10}\} = \{-69.8675, 40.6365, -76.1113, 65.0137, -93.6473, 92.6166, 80.1439, 76.2718, 26.7529, 37.0511\} \\
& \mathbf{v}_3 = \{v_3^1, \ldots, v_3^{10}\} = \{46.3398, 57.3869, -59.6658, 54.2175, -66.4293, 23.2850, 8.9172, 5.8984, -81.7778, -29.9939\} \\
& \mathbf{v}_4 = \{v_4^1, \ldots, v_4^{10}\} = \{-94.7804, 90.0091, -12.3052, 27.1707, 89.5608, 23.1004, -82.2399, 22.3617, 87.6216, -60.2491\} \\
& \mathbf{v}_5 = \{v_5^1, \ldots, v_5^{10}\} = \{-14.1227, 70.3685, -3.5821, -57.3306, -21.0553, 22.6570, -8.0322, 66.9846, 22.4542, -21.2894\} \\
& \mathbf{v}_6 = \{v_6^1, \ldots, v_6^{10}\} = \{-22.2035, -94.6947, -28.3114, 79.0958, -32.9711, -68.5177, -37.1964, 49.1284, -50.3006, \\
& \qquad 31.9762\} \\
& \mathbf{v}_7 = \{v_7^1, \ldots, v_7^{10}\} = \{63.0211, 15.4585, -0.9510, 90.1246, 18.4635, -17.3490, 96.5306, -14.2523, -37.9036, 58.6272\} \\
& \mathbf{v}_8 = \{v_8^1, \ldots, v_8^{10}\} = \{-96.1546, -3.5673, -59.5396, 13.2944, -21.4157, 25.7777, 0.5632, 73.3501, -29.0539, -79.8143\} \\
& x^{cn}_1, \ldots, x^{cn}_2 \in [-100, 100] \\
& x^{ca}_j \in \mathbf{v}_j, j \in \{1, 2, 3, 4, 5, 6, 7, 8\}
\end{aligned}
$$

where A is an orthogonal matrix. The optimal solution is: $\mathbf{x}^{cn}_{best} = (6.5706, -4.7415)$ and $\mathbf{x}^{ca}_{best} = (99.8101, -69.8675, 46.3398, -94.7804, -14.1227, -22.2035, 63.0211, -96.1546)$, and $f(\mathbf{x}^{cn}_{best}, \mathbf{x}^{ca}_{best}) = 0$.

$$
\begin{aligned}
\mathbf{F20}: \ min: \ & f(\mathbf{x}^{cn}, \mathbf{x}^{ca}) = 1 + \sum_{i=1}^{10} \frac{z_i^2}{4000} - \prod_{i=1}^{10} \cos\left(\frac{z_i}{\sqrt{i}}\right) \\
& \mathbf{z} = (z_1, \ldots, z_{10}) = 6 * \mathbf{z}' A \\
& z'_i = \begin{cases} x^{cn}_i - o_i, & i \in \{1, \ldots, 2\} \\ x^{ca}_{i-2} - o_i, & i \in \{3, \ldots, 10\} \end{cases} \\
& \mathbf{o} = (o_1, \ldots, o_{10}) = (84.9982, 65.1519, -12.0721, -10.4880, 2.4412, -86.8010, -32.6664, -39.7689, 39.6034, 52.0954) \\
& \mathbf{v}_1 = \{v_1^1, \ldots, v_1^{10}\} = \{-12.0721, 2.9979, -19.0989, -67.9707, -5.3256, 41.2393, 88.4943, -89.4662, 12.4275, 53.2186\} \\
& \mathbf{v}_2 = \{v_2^1, \ldots, v_2^{10}\} = \{-10.4880, 49.2424, 64.1867, 5.7290, -95.5992, 66.8645, 56.9595, -27.3189, 77.8113, -53.0569\} \\
& \mathbf{v}_3 = \{v_3^1, \ldots, v_3^{10}\} = \{2.4412, 63.1647, -11.6669, -31.9233, -87.5237, 48.4239, -75.0023, 49.5995, -83.9520, \\
& \qquad -81.1888\} \\
& \mathbf{v}_4 = \{v_4^1, \ldots, v_4^{10}\} = \{-86.8010, 58.9903, -71.2887, 55.7150, 59.1220, 37.2548, 75.7530, -10.7222, -1.7561, 97.9166\} \\
& \mathbf{v}_5 = \{v_5^1, \ldots, v_5^{10}\} = \{-32.6664, 14.3942, 19.6051, 23.8658, 92.7302, 0.4261, 19.6791, 60.4852, -39.3893, -35.6968\} \\
& \mathbf{v}_6 = \{v_6^1, \ldots, v_6^{10}\} = \{-39.7689, 51.2260, 95.1572, -52.9366, 81.0941, -67.8047, -13.8984, 74.2628, 41.1879, 53.5652\} \\
& \mathbf{v}_7 = \{v_7^1, \ldots, v_7^{10}\} = \{39.6034, -98.0900, 66.9081, 5.6974, -52.6892, -22.8678, -21.4237, -70.5337, 8.6142, -89.3348\} \\
& \mathbf{v}_8 = \{v_8^1, \ldots, v_8^{10}\} = \{52.0954, -95.0943, 60.2757, -59.7487, -89.1344, 70.3986, -55.3637, 17.6185, -72.3865, \\
& \qquad -10.6520\} \\
& x^{cn}_1, \ldots, x^{cn}_2 \in [-100, 100] \\
& x^{ca}_j \in \mathbf{v}_j, j \in \{1, 2, 3, 4, 5, 6, 7, 8\}
\end{aligned}
$$

where A is an orthogonal matrix. The optimal solution is: $\mathbf{x}^{cn}_{best} = (84.9982, 65.1519)$ and $\mathbf{x}^{ca}_{best} = (-12.0721, -10.4880, 2.4412,$

$-86.8010, -32.6664, -39.7689, 39.6034, 52.0954)$, and $f(\mathbf{x}^{cn}_{best}, \mathbf{x}^{ca}_{best}) = 0$.

$$
\begin{aligned}
\mathbf{F21}: \ min: \ & f(\mathbf{x}^{cn}, \mathbf{x}^{ca}) = \sum_{i=1}^{10} z_i^2 \\
& \mathbf{z} = (z_1, \ldots, z_{10}) = \mathbf{z}' A \\
& z'_i = \begin{cases} x_i^{cn} - o_i, & i \in \{1, \ldots, 5\} \\ x_{i-5}^{ca} - o_i, & i \in \{6, \ldots, 10\} \end{cases} \\
& \mathbf{o} = (o_1, \ldots, o_{10}) = (7.7624, -51.0984, -95.5110, -68.7425, 8.7344, 0.0577, -36.7734, 44.3837, 99.8131, -12.1793) \\
& \mathbf{v}_1 = \{v_1^1, \ldots, v_1^5\} = \{0.0577, -66.8891, -24.5506, -96.2061, 45.4579\} \\
& \mathbf{v}_2 = \{v_2^1, \ldots, v_2^5\} = \{-36.7734, 11.1348, 40.9187, -32.3377, 62.3757\} \\
& \mathbf{v}_3 = \{v_3^1, \ldots, v_3^5\} = \{44.3837, -84.2635, -31.8857, -99.0299, 23.2041\} \\
& \mathbf{v}_4 = \{v_4^1, \ldots, v_4^5\} = \{99.8131, 38.7794, 97.4385, 66.3214, 83.6572\} \\
& \mathbf{v}_5 = \{v_5^1, \ldots, v_5^5\} = \{-12.1793, -81.4490, 94.5925, -20.7460, -23.4447\} \\
& x_1^{cn}, \ldots, x_5^{cn} \in [-100, 100] \\
& x_j^{ca} \in \mathbf{v}_j, j \in \{1, 2, 3, 4, 5\}
\end{aligned}
$$

where A is an orthogonal matrix. The optimal solution is: $\mathbf{x}^{cn}_{best} = (7.7624, -51.0984, -95.5110, -68.7425, 8.7344)$ and $\mathbf{x}^{ca}_{best} = (0.0577, -36.7734, 44.3837, 99.8131, -12.1793)$, and $f(\mathbf{x}^{cn}_{best}, \mathbf{x}^{ca}_{best}) = 0$.

$$
\begin{aligned}
\mathbf{F22}: \ min: \ & f(\mathbf{x}^{cn}, \mathbf{x}^{ca}) = \sum_{i=1}^{10} (z_i^2 - 10cos(2\pi z_i) + 10) \\
& \mathbf{z} = (z_1, \ldots, z_{10}) = 0.05 * \mathbf{z}' A \\
& z'_i = \begin{cases} x_i^{cn} - o_i, & i \in \{1, \ldots, 5\} \\ x_{i-5}^{ca} - o_i, & i \in \{6, \ldots, 10\} \end{cases} \\
& \mathbf{o} = (o_1, \ldots, o_{10}) = (0.5876, -84.9703, -97.8543, 39.7223, 28.3686, 61.1286, 38.9558, -77.1346, 50.6776, 14.4813) \\
& \mathbf{v}_1 = \{v_1^1, \ldots, v_1^5\} = \{61.1286, 4.1286, -3.9445, -15.1705, -7.9428\} \\
& \mathbf{v}_2 = \{v_2^1, \ldots, v_2^5\} = \{38.9558, 37.3101, 83.0975, -98.6905, -75.3426\} \\
& \mathbf{v}_3 = \{v_3^1, \ldots, v_3^5\} = \{-77.1346, -2.0340, 70.8802, 51.3176, -24.3460\} \\
& \mathbf{v}_4 = \{v_4^1, \ldots, v_4^5\} = \{50.6776, -39.6234, -61.7100, 97.7223, 63.1775\} \\
& \mathbf{v}_5 = \{v_5^1, \ldots, v_5^5\} = (14.4813, -97.4609, 92.2885, -3.8172, 83.2134\} \\
& x_1^{cn}, \ldots, x_5^{cn} \in [-100, 100] \\
& x_j^{ca} \in \mathbf{v}_j, j \in \{1, 2, 3, 4, 5\}
\end{aligned}
$$

where A is an orthogonal matrix. The optimal solution is: $\mathbf{x}^{cn}_{best} = (0.5876, -84.9703, -97.8543, 39.7223, 28.3686)$ and $\mathbf{x}^{ca}_{best} =$

$(61.1286, 38.9558, -77.1346, 50.6776, 14.4813)$, and $f(\mathbf{x}^{cn}_{best}, \mathbf{x}^{ca}_{best}) = 0$.

$$
\begin{aligned}
\mathbf{F23}: \ min: \ & f(\mathbf{x}^{cn}, \mathbf{x}^{ca}) = 20 + e - 20\exp(-0.2\sqrt{\frac{1}{10}\sum_{i=1}^{10} z_i^2}) - \exp(\frac{1}{10}\sum_{i=1}^{10}\cos(2\pi z_i)) \\
& \mathbf{z} = (z_1, \ldots, z_{10}) = 0.32 * \mathbf{z}' A \\
& z'_i = \begin{cases} x_i^{cn} - o_i, & i \in \{1, \ldots, 5\} \\ x_{i-5}^{ca} - o_i, & i \in \{6, \ldots, 10\} \end{cases} \\
& \mathbf{o} = (o_1, \ldots, o_{10}) = (5.3830, 0.1560, 47.8659, -17.0994, -32.5756, -29.2208, -32.7262, -43.5349, 1.9141, -36.7252) \\
& \mathbf{v}_1 = \{v_1^1, \ldots, v_1^5\} = \{-29.2208, -14.6371, 31.1216, -14.0404, 23.6011\} \\
& \mathbf{v}_2 = \{v_2^1, \ldots, v_2^5\} = \{-32.7262, 53.3853, -56.2406, 51.0570, 21.1575\} \\
& \mathbf{v}_3 = \{v_3^1, \ldots, v_3^5\} = \{-43.5349, -59.4140, -49.4245, -9.3890, -56.9489\} \\
& \mathbf{v}_4 = \{v_4^1, \ldots, v_4^5\} = \{1.9141, -12.6618, -3.5678, -18.1508, -7.8900\} \\
& \mathbf{v}_5 = \{v_5^1, \ldots, v_5^5\} = \{-36.7252, 27.8390, -53.1350, -21.6579, -27.2089\} \\
& x_1^{cn}, \ldots, x_5^{cn} \in [-100, 100] \\
& x_j^{ca} \in \mathbf{v}_j, j \in \{1, 2, 3, 4, 5\}
\end{aligned}
$$

where A is an orthogonal matrix. The optimal solution is $\mathbf{x}^{cn}_{best} = (5.3830, 0.1560, 47.8659, -17.0994, -32.5756)$ and $\mathbf{x}^{ca}_{best} = (-29.2208, -32.7262, -43.5349, 1.9141, -36.7252)$, and $f(\mathbf{x}^{cn}_{best}, \mathbf{x}^{ca}_{best}) = 0$.

$$
\begin{aligned}
\mathbf{F24}: \ min: \ & f(\mathbf{x}^{cn}, \mathbf{x}^{ca}) = \sum_{i=1}^{10} i x_i^2 \\
& \mathbf{z} = (z_1, \ldots, z_{10}) = 0.05 * \mathbf{z}' A \\
& z'_i = \begin{cases} x_i^{cn} - o_i, & i \in \{1, \ldots, 5\} \\ x_{i-5}^{ca} - o_i, & i \in \{6, \ldots, 10\} \end{cases} \\
& \mathbf{o} = (o_1, \ldots, o_{10}) = (6.5706, -4.7415, 99.8101, -69.8675, 46.3398, -94.7804, -14.1227, -22.2035, 63.0211, -96.1546) \\
& \mathbf{v}_1 = \{v_1^1, \ldots, v_1^5\} = \{-94.7804, 90.0091, -12.3052, 27.1707, 89.5608\} \\
& \mathbf{v}_2 = \{v_2^1, \ldots, v_2^5\} = \{-14.1227, 70.3685, -3.5821, -57.3306, -21.0553\} \\
& \mathbf{v}_3 = \{v_3^1, \ldots, v_3^5\} = \{-22.2035, -94.6947, -28.3114, 79.0958, -32.9711\} \\
& \mathbf{v}_4 = \{v_4^1, \ldots, v_4^5\} = \{63.0211, 15.4585, -0.9510, 90.1246, 18.4635\} \\
& \mathbf{v}_5 = \{v_5^1, \ldots, v_5^5\} = \{-96.1546, -3.5673, -59.5396, 13.2944, -21.4157\} \\
& x_1^{cn}, \ldots, x_5^{cn} \in [-100, 100] \\
& x_j^{ca} \in \mathbf{v}_j, j \in \{1, 2, 3, 4, 5\}
\end{aligned}
$$

where A is an orthogonal matrix. The optimal solution is: $\mathbf{x}^{cn}_{best} = (6.5706, -4.7415, 99.8101, -69.8675, 46.3398)$ and $\mathbf{x}^{ca}_{best} =$

$(-94.7804, -14.1227, -22.2035, 63.0211, -96.1546)$, and $f(\mathbf{x}^{cn}_{best}, \mathbf{x}^{ca}_{best}) = 0$.

$$
\begin{aligned}
\textbf{F25}:\ min:\ & f(\mathbf{x}^{cn}, \mathbf{x}^{ca}) = 1 + \sum_{i=1}^{10} \frac{z_i^2}{4000} - \prod_{i=1}^{10} \cos(\frac{z_i}{\sqrt{i}}) \\
& \mathbf{z} = (z_1, \ldots, z_{10}) = 6 * \mathbf{z}' A \\
& z'_i = \begin{cases} x_i^{cn} - o_i, & i \in \{1, \ldots, 5\} \\ x_{i-5}^{ca} - o_i, & i \in \{6, \ldots, 10\} \end{cases} \\
& \mathbf{o} = (o_1, \ldots, o_{10}) = (84.9982, 65.1519, -12.0721, -10.4880, 2.4412, -86.8010, -32.6664, -39.7689, 39.6034, 52.0954) \\
& \mathbf{v}_1 = \{v_1^1, \ldots, v_1^5\} = \{-86.8010, 58.9903, -71.2887, 55.7150, 59.1220\} \\
& \mathbf{v}_2 = \{v_2^1, \ldots, v_2^5\} = \{-32.6664, 14.3942, 19.6051, 23.8658, 92.7302\} \\
& \mathbf{v}_3 = \{v_3^1, \ldots, v_3^5\} = \{-39.7689, 51.2260, 95.1572, -52.9366, 81.0941\} \\
& \mathbf{v}_4 = \{v_4^1, \ldots, v_4^5\} = \{39.6034, -98.0900, 66.9081, 5.6974, -52.6892\} \\
& \mathbf{v}_5 = \{v_5^1, \ldots, v_5^5\} = \{52.0954, -95.0943, 60.2757, -59.7487, -89.1344\} \\
& x_1^{cn}, \ldots, x_5^{cn} \in [-100, 100] \\
& x_j^{ca} \in \mathbf{v}_j, j \in \{1, 2, 3, 4, 5\}
\end{aligned}
$$

where A is an orthogonal matrix. The optimal solution is: $\mathbf{x}^{cn}_{best} = (84.9982, 65.1519, -12.0721, -10.4880, 2.4412)$ and $\mathbf{x}^{ca}_{best} = (-86.8010, -32.6664, -39.7689, 39.6034, 52.0954)$, and $f(\mathbf{x}^{cn}_{best}, \mathbf{x}^{ca}_{best}) = 0$.

$$
\begin{aligned}
\textbf{F26}:\ min:\ & f(\mathbf{x}^{cn}, \mathbf{x}^{ca}) = \sum_{i=1}^{10} z_i^2 \\
& \mathbf{z} = (z_1, \ldots, z_{10}) = \mathbf{z}' A \\
& z'_i = \begin{cases} x_i^{cn} - o_i, & i \in \{1, \ldots, 5\} \\ x_{i-5}^{ca} - o_i, & i \in \{6, \ldots, 10\} \end{cases} \\
& \mathbf{o} = (o_1, \ldots, o_{10}) = (7.7624, -51.0984, -95.5110, -68.7425, 8.7344, 0.0577, -36.7734, 44.3837, 99.8131, -12.1793) \\
& \mathbf{v}_1 = \{v_1^1, \ldots, v_1^{10}\} = \{0.0577, -66.8891, -24.5506, -96.2061, 45.4579, 18.1319, 58.3241, 71.9285, -75.8105, -22.9862\} \\
& \mathbf{v}_2 = \{v_2^1, \ldots, v_2^{10}\} = \{-36.7734, 11.1348, 40.9187, -32.3377, 62.3757, 24.4644, 86.2232, 90.6468, -99.4558, 89.3221\} \\
& \mathbf{v}_3 = \{v_3^1, \ldots, v_3^{10}\} = \{44.3837, -84.2635, -31.8857, -99.0299, 23.2041, 13.1996, -43.5760, 26.1334, -6.6750, -22.8134\} \\
& \mathbf{v}_4 = \{v_4^1, \ldots, v_4^{10}\} = \{99.8131, 38.7794, 97.4385, 66.3214, 83.6572, 64.3900, 6.2714, 4.7893, 42.9978, -46.2021\} \\
& \mathbf{v}_5 = \{v_5^1, \ldots, v_5^{10}\} = \{-12.1793, -81.4490, 94.5925, -20.7460, -23.4447, -37.0443, -33.7724, -78.8025, 69.8750, \\
& \qquad 70.8563\} \\
& x_1^{cn}, \ldots, x_5^{cn} \in [-100, 100] \\
& x_j^{ca} \in \mathbf{v}_j, j \in \{1, 2, 3, 4, 5\}
\end{aligned}
$$

where A is an orthogonal matrix. The optimal solution is: $\mathbf{x}^{cn}_{best} = (7.7624, -51.0984, -95.5110, -68.7425, 8.7344)$ and $\mathbf{x}^{ca}_{best} =$

$(0.0577, -36.7734, 44.3837, 99.8131, -12.1793)$, and $f(\mathbf{x}_{best}^{cn}, \mathbf{x}_{best}^{ca}) = 0$.

$$\mathbf{F27}: \; min: \; f(\mathbf{x}^{cn}, \mathbf{x}^{ca}) = \sum_{i=1}^{10}(z_i^2 - 10cos(2\pi z_i) + 10)$$

$$\mathbf{z} = (z_1, \ldots, z_{10}) = 0.05 * \mathbf{z}'A$$

$$z_i' = \begin{cases} x_i^{cn} - o_i, & i \in \{1, \ldots, 5\} \\ x_{i-5}^{ca} - o_i, & i \in \{6, \ldots, 10\} \end{cases}$$

$$\mathbf{o} = (o_1, \ldots, o_{10}) = (0.5876, -84.9703, -97.8543, 39.7223, 28.3686, 61.1286, 38.9558, -77.1346, 50.6776, 14.4813)$$

$$\mathbf{v}_1 = \{v_1^1, \ldots, v_1^{10}\} = \{61.1286, 4.1286, -3.9445, -15.1705, -7.9428, -51.8438, 76.1710, -37.0110, -34.0411, -61.8862\}$$

$$\mathbf{v}_2 = \{v_2^1, \ldots, v_2^{10}\} = \{38.9558, 37.3101, 83.0975, -98.6905, -75.3426, 93.3490, 52.6871, -77.6949, 3.4488, -14.2671\}$$

$$\mathbf{v}_3 = \{v_3^1, \ldots, v_3^{10}\} = \{-77.1346, -2.0340, 70.8802, 51.3176, -24.3460, 63.8007, 93.8858, 11.8351, 57.5630, -0.9117\}$$

$$\mathbf{v}_4 = \{v_4^1, \ldots, v_4^{10}\} = \{50.6776, -39.6234, -61.7100, 97.7223, 63.1775, -12.0348, -21.6271, -12.3744, 67.1491, -19.0775\}$$

$$\mathbf{v}_5 = \{v_5^1, \ldots, v_5^{10}\} = \{14.4813, -97.4609, 92.2885, -3.8172, 83.2134, -89.4358, 10.1637, -86.6364, -64.1289, 6.0189\}$$

$$x_1^{cn}, \ldots, x_5^{cn} \in [-100, 100]$$

$$x_j^{ca} \in \mathbf{v}_j, j \in \{1, 2, 3, 4, 5\}$$

where A is an orthogonal matrix. The optimal solution is: $\mathbf{x}_{best}^{cn} = (0.5876, -84.9703, -97.8543, 39.7223, 28.3686)$ and $\mathbf{x}_{best}^{ca} = (61.1286, 38.9558, -77.1346, 50.6776, 14.4813)$, and $f(\mathbf{x}_{best}^{cn}, \mathbf{x}_{best}^{ca}) = 0$.

$$\mathbf{F28}: \; min: \; f(\mathbf{x}^{cn}, \mathbf{x}^{ca}) = 20 + e - 20\exp(-0.2\sqrt{\frac{1}{10}\sum_{i=1}^{10} z_i^2}) - \exp(\frac{1}{10}\sum_{i=1}^{10}\cos(2\pi z_i))$$

$$\mathbf{z} = (z_1, \ldots, z_{10}) = 0.32 * \mathbf{z}'A$$

$$z_i' = \begin{cases} x_i^{cn} - o_i, & i \in \{1, \ldots, 5\} \\ x_{i-5}^{ca} - o_i, & i \in \{6, \ldots, 10\} \end{cases}$$

$$\mathbf{o} = (o_1, \ldots, o_{10}) = (5.3830, 0.1560, 47.8659, -17.0994, -32.5756, -29.2208, -32.7262, -43.5349, 1.9141, -36.7252)$$

$$\mathbf{v}_1 = \{v_1^1, \ldots, v_1^{10}\} = \{-29.2208, -14.6371, 31.1216, -14.0404, 23.6011, -27.5998, 50.6179, 26.2418, 43.5885, 15.1259\}$$

$$\mathbf{v}_2 = \{v_2^1, \ldots, v_2^{10}\} = \{-32.7262, 53.3853, -56.2406, 51.0570, 21.1575, 20.4656, -28.2398, -13.7042, -18.0062, 10.0906\}$$

$$\mathbf{v}_3 = \{v_3^1, \ldots, v_3^{10}\} = \{-43.5349, -59.4140, -49.4245, -9.3890, -56.9489, -50.0486, -53.9074, -58.2566, 17.3949, -27.1704\}$$

$$\mathbf{v}_4 = \{v_4^1, \ldots, v_4^{10}\} = \{1.9141, -12.6618, -3.5678, -18.1508, -7.8900, 1.7955, -45.1022, -14.7915, -47.5095, -57.2121\}$$

$$\mathbf{v}_5 = \{v_5^1, \ldots, v_5^{10}\} = \{-36.7252, 27.8390, -53.1350, -21.6579, -27.2089, -58.0376, 19.1243, 2.8412, -17.4512, 58.3012\}$$

$$x_1^{cn}, \ldots, x_5^{cn} \in [-100, 100]$$

$$x_j^{ca} \in \mathbf{v}_j, j \in \{1, 2, 3, 4, 5\}$$

where A is an orthogonal matrix. The optimal solution is: $\mathbf{x}_{best}^{cn} = (5.3830, 0.1560, 47.8659, -17.0994, -32.5756)$ and $\mathbf{x}_{best}^{ca} =$

$(-29.2208, -32.7262, -43.5349, 1.9141, -36.7252)$, and $f(\mathbf{x}^{cn}_{best}, \mathbf{x}^{ca}_{best}) = 0$.

$$\mathbf{F29}: \; min: \; f(\mathbf{x}^{cn}, \mathbf{x}^{ca}) = \sum_{i=1}^{10} i x_i^2$$
$$\mathbf{z} = (z_1, \ldots, z_{10}) = 0.05 * \mathbf{z}' A$$
$$z_i' = \begin{cases} x_i^{cn} - o_i, & i \in \{1, \ldots, 5\} \\ x_{i-5}^{ca} - o_i, & i \in \{6, \ldots, 10\} \end{cases}$$
$$\mathbf{o} = (o_1, \ldots, o_{10}) = (6.5706, -4.7415, 99.8101, -69.8675, 46.3398, -94.7804, -14.1227, -22.2035, 63.0211, -96.1546)$$
$$\mathbf{v}_1 = \{v_1^1, \ldots, v_1^{10}\} = \{-94.7804, 90.0091, -12.3052, 27.1707, 89.5608, 23.1004, -82.2399, 22.3617, 87.6216, -60.2491\}$$
$$\mathbf{v}_2 = \{v_2^1, \ldots, v_2^{10}\} = \{-14.1227, 70.3685, -3.5821, -57.3306, -21.0553, 22.6570, -8.0322, 66.9846, 22.4542, -21.2894\}$$
$$\mathbf{v}_3 = \{v_3^1, \ldots, v_3^{10}\} = \{-22.2035, -94.6947, -28.3114, 79.0958, -32.9711, -68.5177, -37.1964, 49.1284, -50.3006, 31.9762\}$$
$$\mathbf{v}_4 = \{v_4^1, \ldots, v_4^{10}\} = \{63.0211, 15.4585, -0.9510, 90.1246, 18.4635, -17.3490, 96.5306, -14.2523, -37.9036, 58.6272\}$$
$$\mathbf{v}_5 = \{v_5^1, \ldots, v_5^{10}\} = \{-96.1546, -3.5673, -59.5396, 13.2944, -21.4157, 25.7777, 0.5632, 73.3501, -29.0539, -79.8143\}$$
$$x_1^{cn}, \ldots, x_5^{cn} \in [-100, 100]$$
$$x_j^{ca} \in \mathbf{v}_j, j \in \{1, 2, 3, 4, 5\}$$

where A is an orthogonal matrix. The optimal solution is: $\mathbf{x}^{cn}_{best} = (6.5706, -4.7415, 99.8101, -69.8675, 46.3398)$ and $\mathbf{x}^{ca}_{best} = (-94.7804, -14.1227, -22.2035, 63.0211, -96.1546)$, and $f(\mathbf{x}^{cn}_{best}, \mathbf{x}^{ca}_{best}) = 0$.

$$\mathbf{F30}: \; min: \; f(\mathbf{x}^{cn}, \mathbf{x}^{ca}) = 1 + \sum_{i=1}^{10} \frac{z_i^2}{4000} - \prod_{i=1}^{10} \cos(\frac{z_i}{\sqrt{i}})$$
$$\mathbf{z} = (z_1, \ldots, z_{10}) = 6 * \mathbf{z}' A$$
$$z_i' = \begin{cases} x_i^{cn} - o_i, & i \in \{1, \ldots, 5\} \\ x_{i-5}^{ca} - o_i, & i \in \{6, \ldots, 10\} \end{cases}$$
$$\mathbf{o} = (o_1, \ldots, o_{10}) = (84.9982, 65.1519, -12.0721, -10.4880, 2.4412, -86.8010, -32.6664, -39.7689, 39.6034, 52.0954)$$
$$\mathbf{v}_1 = \{v_1^1, \ldots, v_1^{10}\} = \{-86.8010, 58.9903, -71.2887, 55.7150, 59.1220, 37.2548, 75.7530, -10.7222, -1.7561, 97.9166\}$$
$$\mathbf{v}_2 = \{v_2^1, \ldots, v_2^{10}\} = \{-32.6664, 14.3942, 19.6051, 23.8658, 92.7302, 0.4261, 19.6791, 60.4852, -39.3893, -35.6968\}$$
$$\mathbf{v}_3 = \{v_3^1, \ldots, v_3^{10}\} = \{-39.7689, 51.2260, 95.1572, -52.9366, 81.0941, -67.8047, -13.8984, 74.2628, 41.1879, 53.5652\}$$
$$\mathbf{v}_4 = \{v_4^1, \ldots, v_4^{10}\} = \{39.6034, -98.0900, 66.9081, 5.6974, -52.6892, -22.8678, -21.4237, -70.5337, 8.6142, -89.3348\}$$
$$\mathbf{v}_5 = \{v_5^1, \ldots, v_5^{10}\} = \{52.0954, -95.0943, 60.2757, -59.7487, -89.1344, 70.3986, -55.3637, 17.6185, -72.3865, -10.6520\}$$
$$x_1^{cn}, \ldots, x_5^{cn} \in [-100, 100]$$
$$x_j^{ca} \in \mathbf{v}_j, j \in \{1, 2, 3, 4, 5\}$$

where A is an orthogonal matrix. The optimal solution is: $\mathbf{x}^{cn}_{best} = (84.9982, 65.1519, -12.0721, -10.4880, 2.4412)$ and $\mathbf{x}^{ca}_{best} = (-86.8010, -32.6664, -39.7689, 39.6034, 52.0954)$, and $f(\mathbf{x}^{cn}_{best}, \mathbf{x}^{ca}_{best}) = 0$.

B. Capacitated Facility Location Problems

The parameters of the capacitated facility location problems used in this paper were generated as follows: D_j was generated from $U[5, 35]$ (U represents the uniform distribution), C_r was generated from $U[1000, 3000]$, $F_{i,r}$ is generated from $U[300, 1300]$, and $Q_{i,j}$ was generated from $U[0, 1]$. By setting m, n, and s, six capacitated facility location problems, i.e., CFLP1-CFLP6, were generated. The parameter settings of CFLP1-CFLP6 are listed in Table S-XVI.

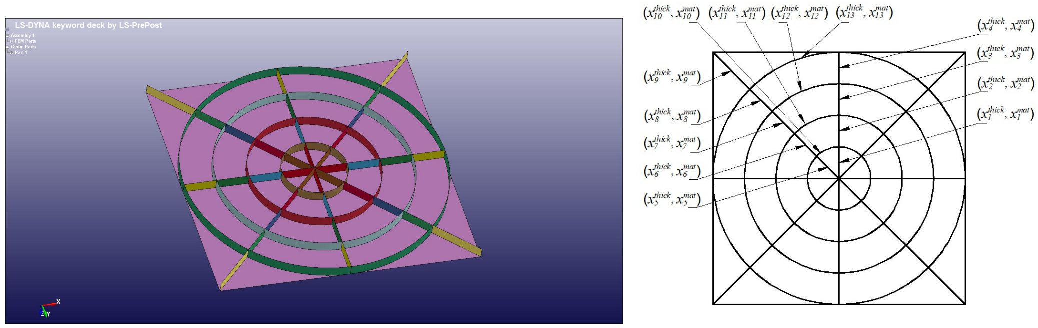
TABLE S-XVI
PARAMETERS SETTINGS OF THE SIX CAPACITATED FACILITY LOCATION PROBLEMS

	m	n	s
CFLP1	5	5	1
CFLP2	10	5	1
CFLP3	5	5	4
CFLP4	10	5	4
CFLP5	5	5	8
CFLP6	10	5	8

C. Dubins Traveling Salesperson Problems

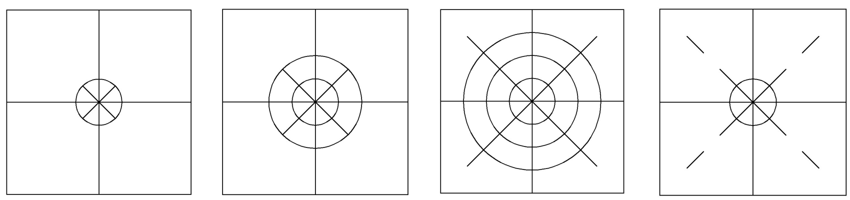
The parameters of the constructed Dubins traveling salesperson problems are set as follows. For DTSP1-DTSP3, the waypoints were randomly generated in the range of $[-50,50]$, and the numbers of waypoints were set to 5, 10, and 15, respectively. For DTSP4-DTSP6, the waypoints were randomly generated in the range of $[-100,100]$, and the numbers of waypoints were also set to 5, 10, and 15, respectively. For all these six problems, the minimal turning radius was set to 1.

S-V. Figures



(a) (b)

Fig. S-1. Structure of the stiffened plate. (a) the considered FEA model (b) the variable distribution of the structure of the stiffened plate.



(a) (b) (c) (d)

Fig. S-2. Topological structures of the stiffened plates reported in [1] and obtained by EGO-Gower, GA, and MiSACO. (a) the original design (b) EGO-Gower (c) GA (d) MiSACO

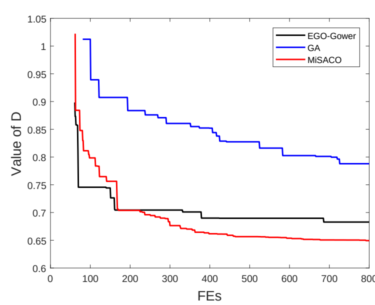


Fig. S-3. Convergence curves derived from EGO-Gower, GA, and MiSACO.

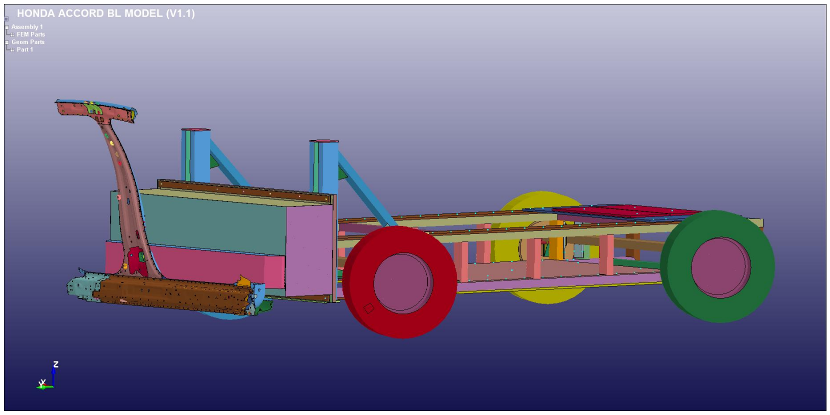


Fig. S-4. Simplified FEA model used in this paper.

References

[1] T. Liu, G. Sun, J. Fang, J. Zhang, and Q. Li, "Topographical design of stiffener layout for plates against blast loading using a modified algorithm," *Structural and Multidisciplinary Optimization*, vol. 59, no. 2, pp. 335–350, 2019.